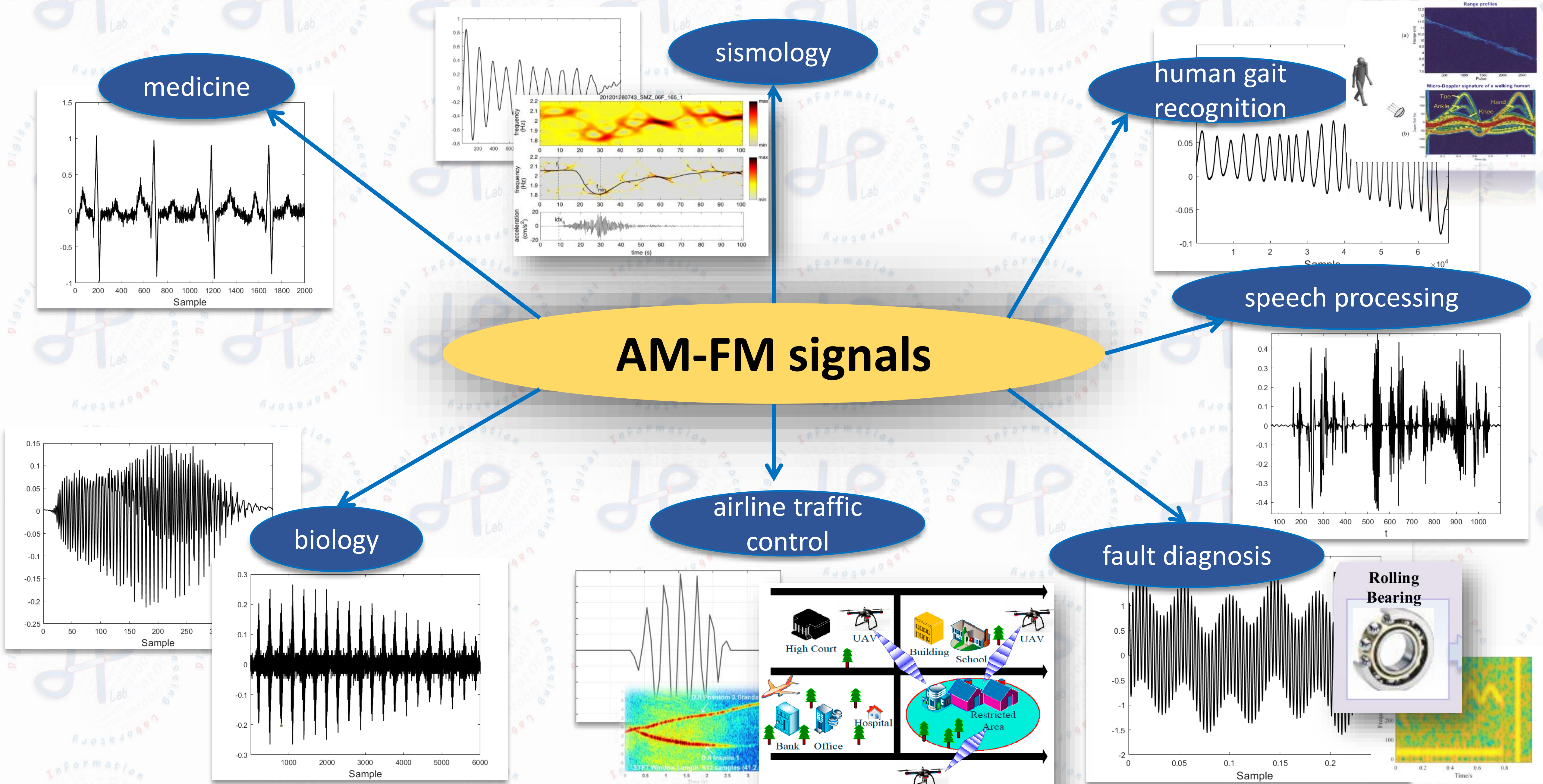
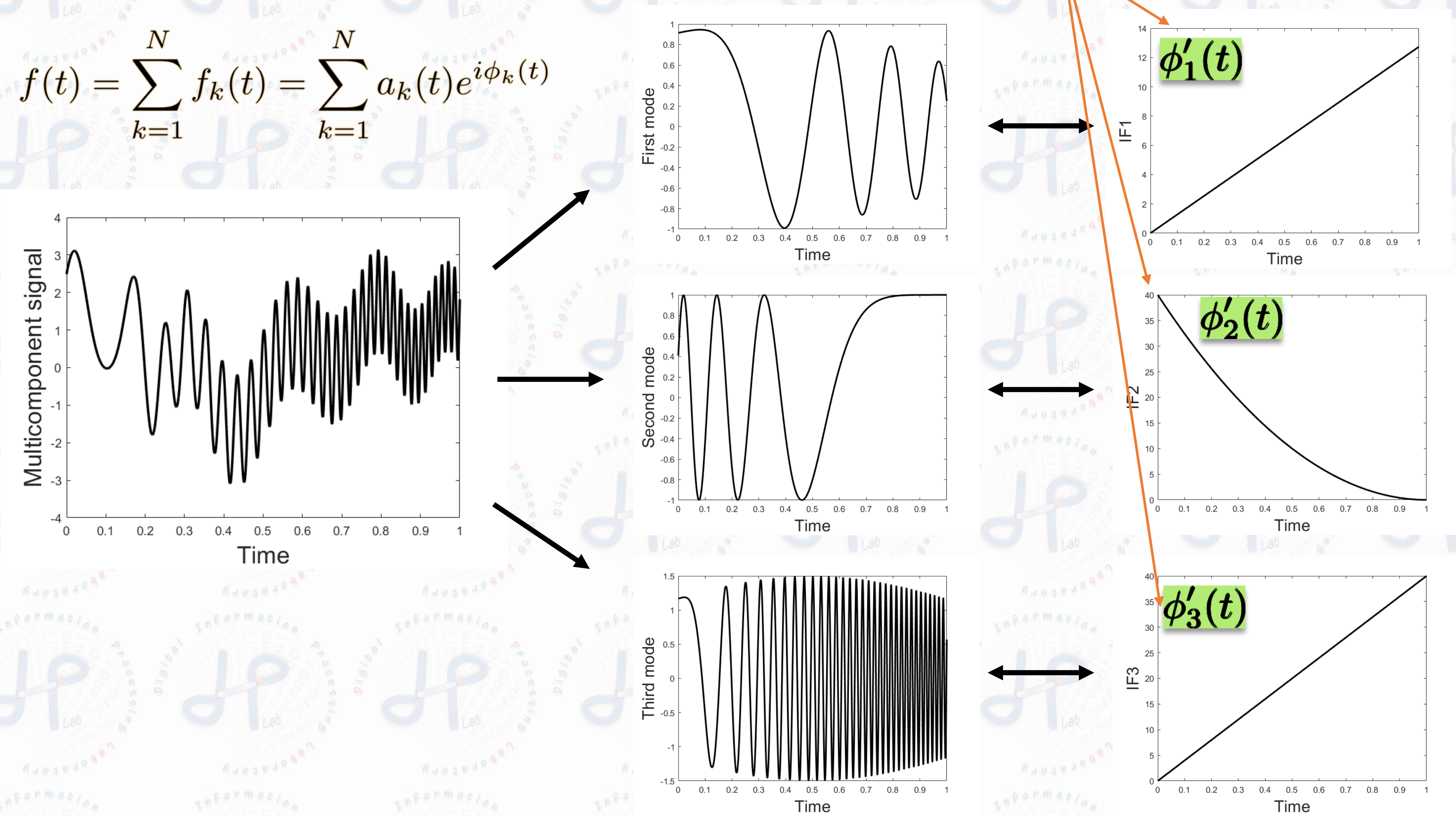
Instantaneous frequency estimation@DIPLab

Amplitude and Frequency Modulated signals everywhere



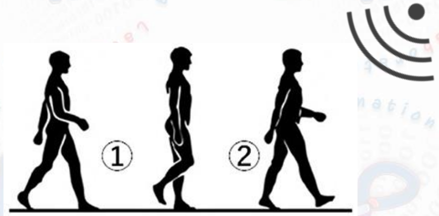
Instantaneous Frequency (IF)

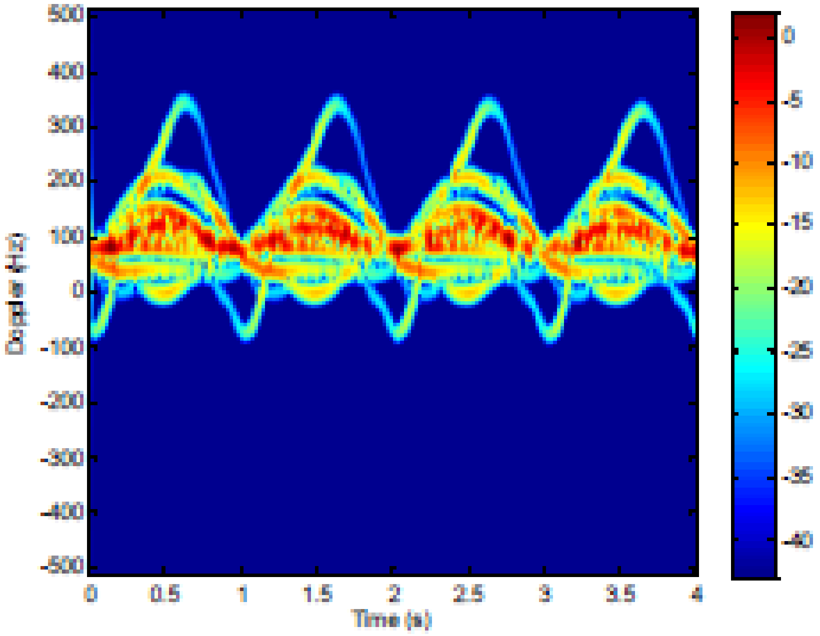
$$f(t) = \sum_{k=1}^{N} f_k(t) = \sum_{k=1}^{N} a_k(t) e^{i\phi_k(t)}$$



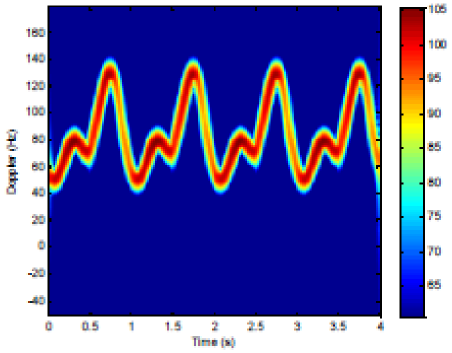
Instantaneous Frequency (IF): human gait recognition

Microdoppler signature of human body: walk

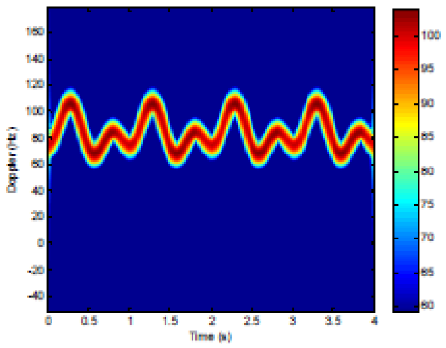




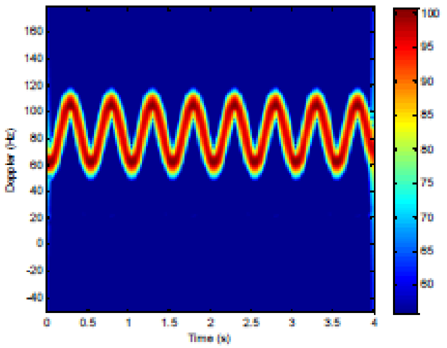
IF = speed of human body components



$\phi_1'(t)$ Leg

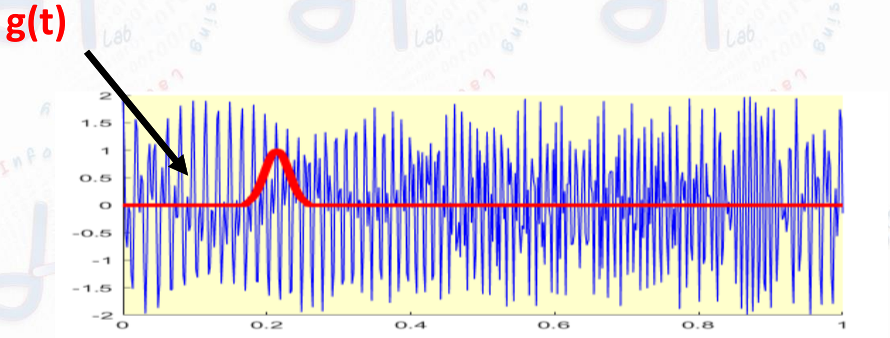


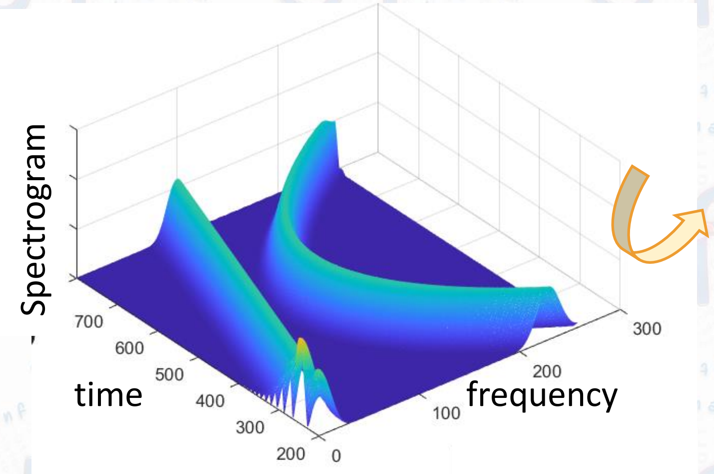
$\phi_2'(t)$ Arm



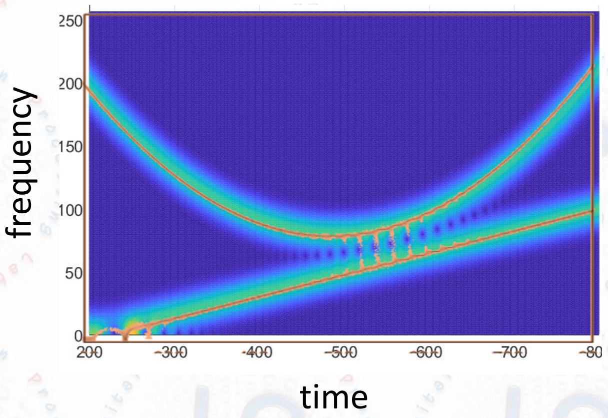
$\phi_3'(t)$ Head

Method: time-frequency analysis



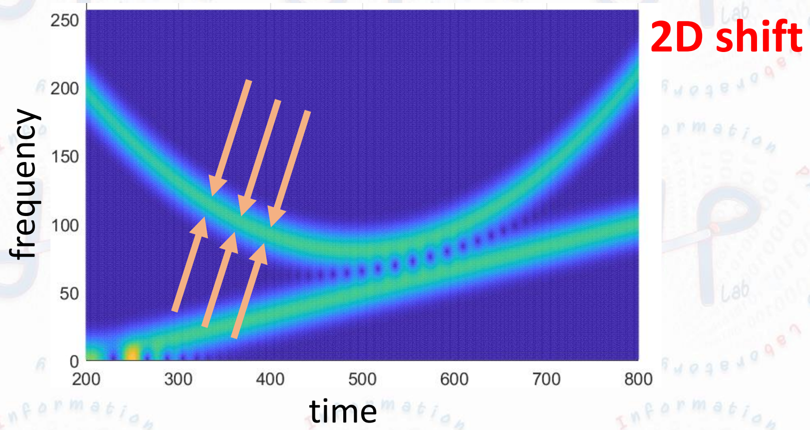


Ridge Curve $(u, \phi'(u))$



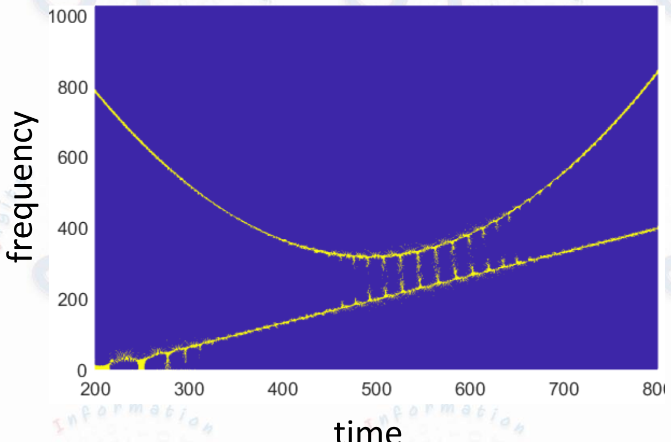
SPECTROGRAM

$$P_S(u, \xi)$$



REASSIGNED SPECTROGRAM

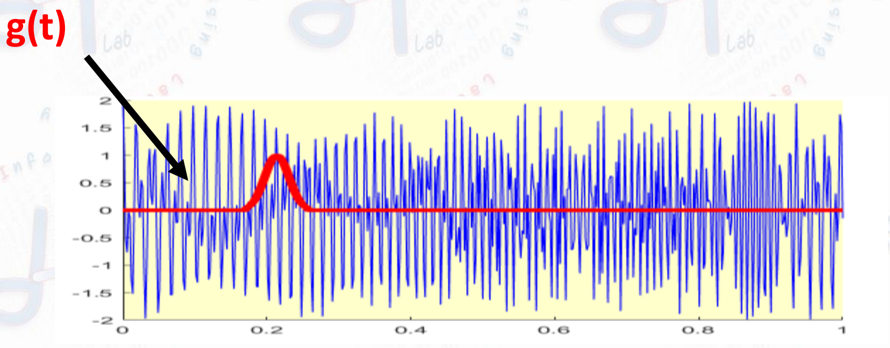
$$\hat{P}_S(u, \xi)$$

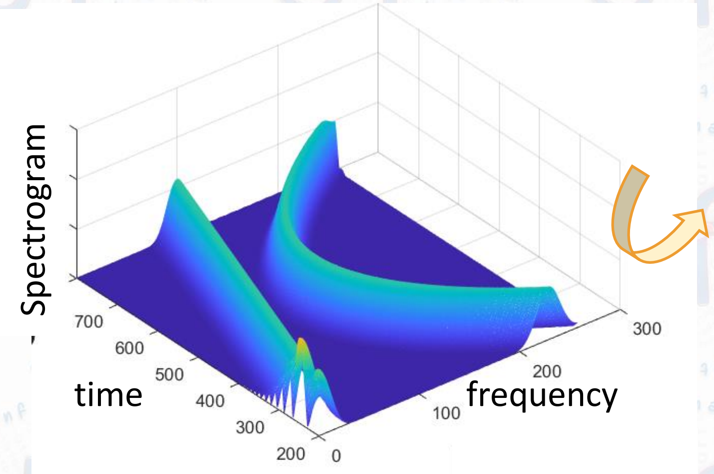


SEPARABILITY CONDITION

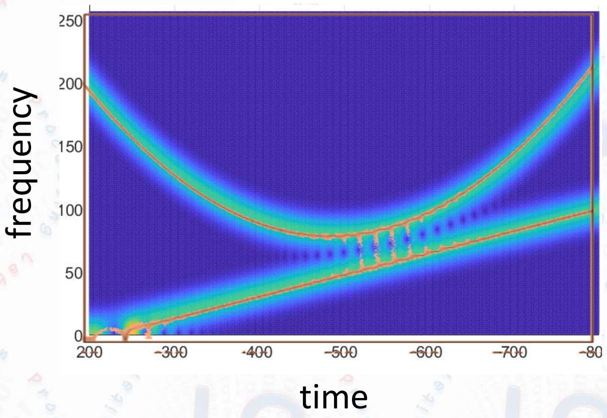
$$|\phi_1'(t) - \phi_2'(t)| \geq \Delta\omega$$

Method: time-frequency analysis



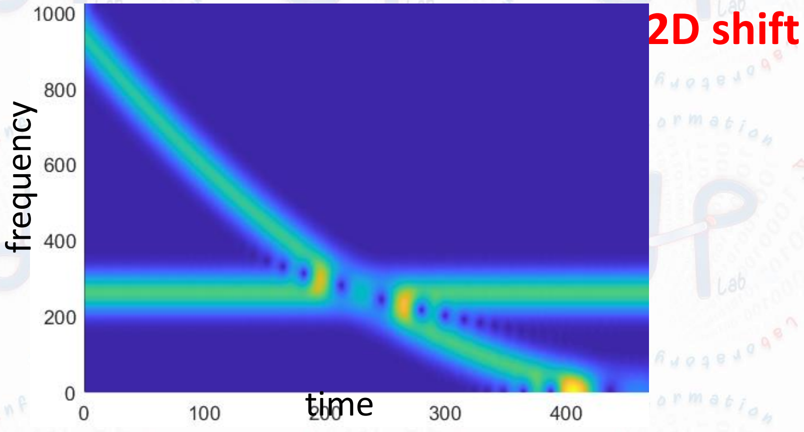


Ridge Curve $(u, \phi'(u))$



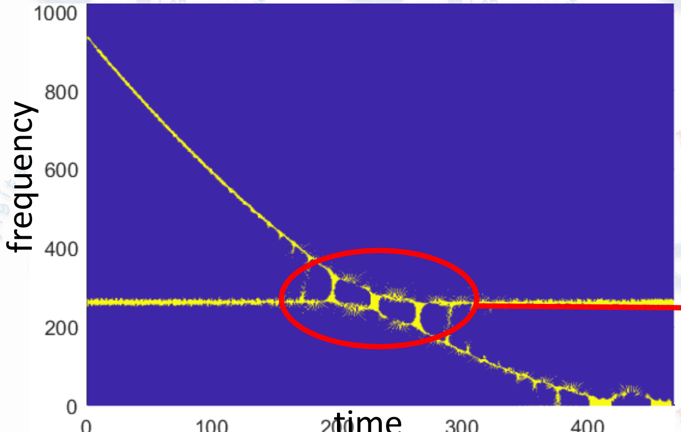
SPECTROGRAM

$$P_S(u, \xi)$$



REASSIGNED SPECTROGRAM

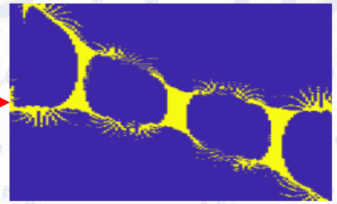
$$\hat{P}_S(u, \xi)$$



SEPARABILITY CONDITION

~~$$|\phi_1'(t) - \phi_2'(t)| \geq \Delta\omega$$~~

$$|\phi_1'(t) - \phi_2'(t)| < \Delta\omega$$



Main contribution

Definition of local and pointwise methods for TF analysis of frequency modulated multicomponent signals having non separable modes

method

spectrogram evolution law and weakened separability

advantages

- ✓ non parametric approaches
- ✓ independency of IF functional class
- ✓ better modes reconstruction in TF interference (non separable) region
- ✓ robustness to moderate noise

requirements

modes counting and *interference region detection*

Weak separability and IF curves estimation

Weak separability

pde transform -based method for **IFs curves reconstruction**

Radon transform-based method for **IFs curves reconstruction**

IF curves recovery: single component

From the spectrogram evolution law:

$$\frac{\partial P(u,\xi)}{\partial u} + \phi''(u)\frac{\partial P(u,\xi)}{\partial \xi} - \frac{2a'(u)}{a(u)}P(u,\xi) = 0$$

to a linear system:

Estimated from the data

unknown

numerical integration $\Rightarrow \phi'(u)$

$$\begin{pmatrix} P_\xi(u,\xi_1) & -P(u,\xi_1) \\ P_\xi(u,\xi_2) & -P(u,\xi_2) \end{pmatrix} \begin{pmatrix} \phi''(u) \\ \frac{2a'(u)}{a(u)} \end{pmatrix} = \begin{pmatrix} -P_u(u,\xi_1) \\ -P_u(u,\xi_2) \end{pmatrix}$$

IF curves recovery: single component

From the spectrogram
evolution law:

$$\frac{\partial P(u,\xi)}{\partial u} + \phi''(u)\frac{\partial P(u,\xi)}{\partial \xi} - \frac{2a'(u)}{a(u)}P(u,\xi) = 0$$

to a linear system:

finite difference approximation

unknown

numerical integration $\Rightarrow$ $\phi'(u)$

$$\begin{pmatrix} P_\xi(u,\xi_1) & -P(u,\xi_1) \\ P_\xi(u,\xi_2) & -P(u,\xi_2) \end{pmatrix} \begin{pmatrix} \phi''(u) \\ \frac{2a'(u)}{a(u)} \end{pmatrix} = \begin{pmatrix} -P_u(u,\xi_1) \\ -P_u(u,\xi_2) \end{pmatrix}$$

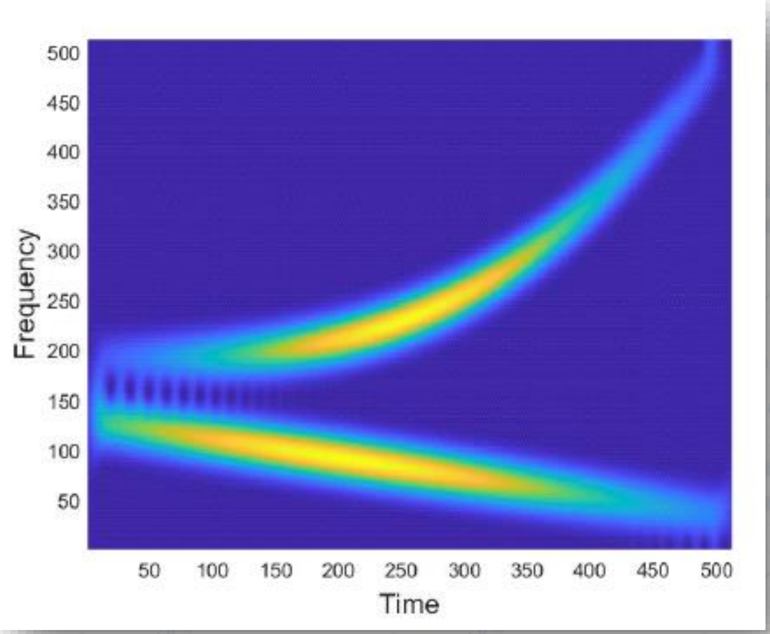
$$P_u(u,\xi) \approx \frac{P(u+h_u,\xi) - P(u-h_u,\xi)}{2h_u},$$

$$P_\xi(u,\xi) \approx \frac{P(u,\xi+h_\xi) - P(u,\xi-h_\xi)}{2h_\xi}, \quad \forall u \in supp\{f\}, \forall \xi > 0,$$

h_u, h_ξ = discretization steps

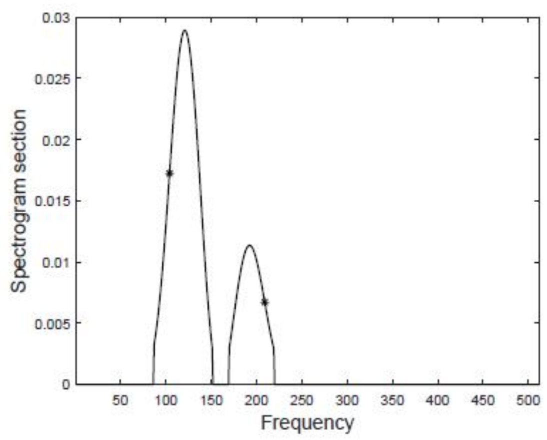
Results

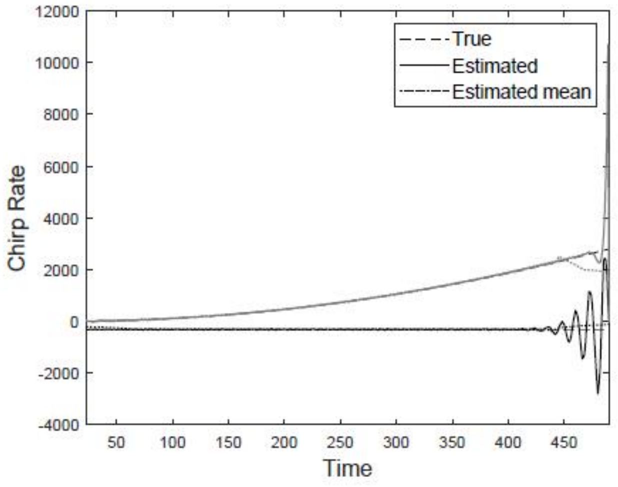
SPECTROGRAM: *gaussian amplitudes and polynomial phases*



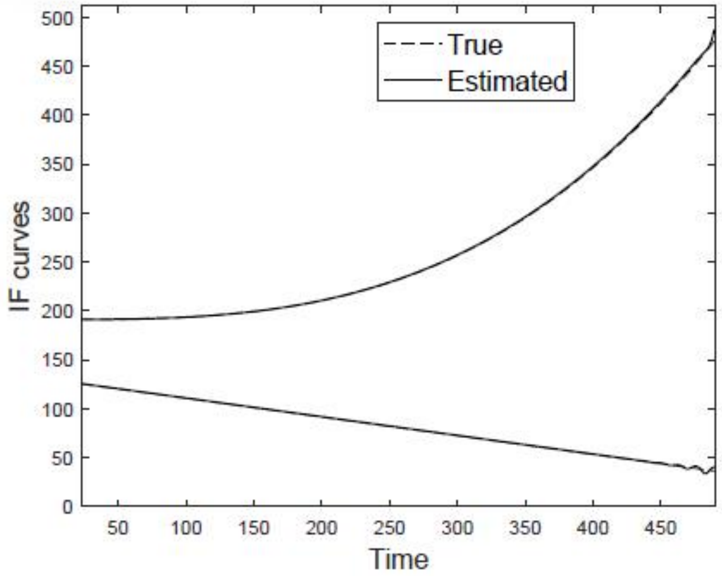
Chirp Rate estimation

Estimation points

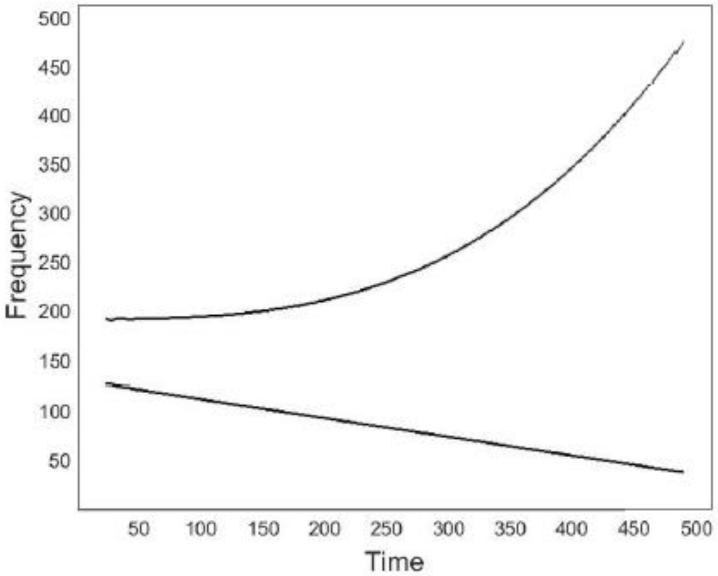




PROPOSED RESULT

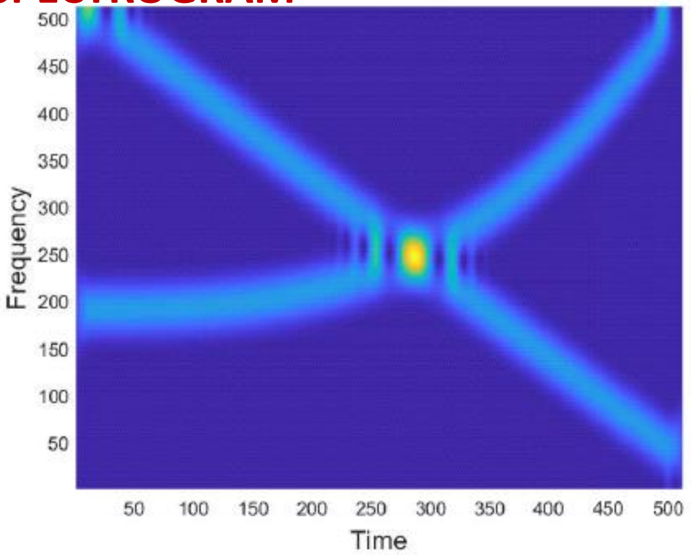


MODULUS MAXIMA

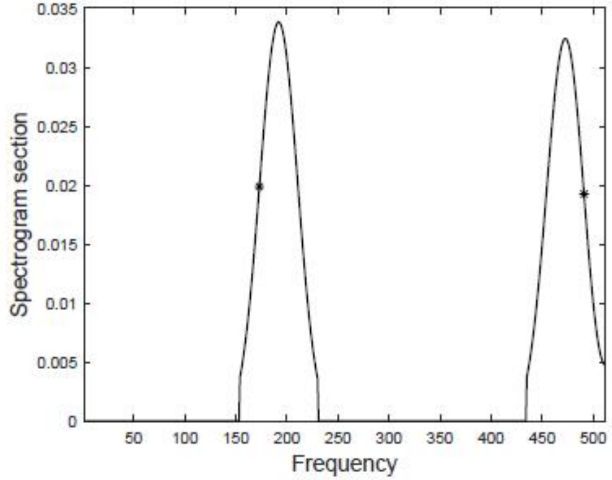


Results

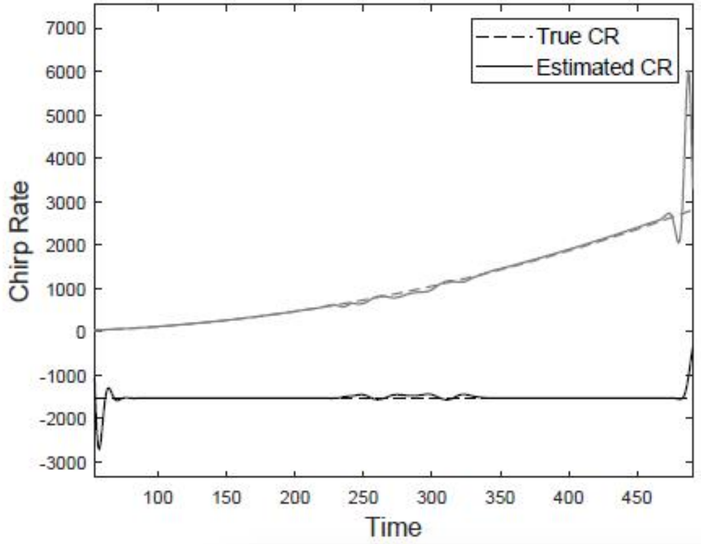
SPECTROGRAM



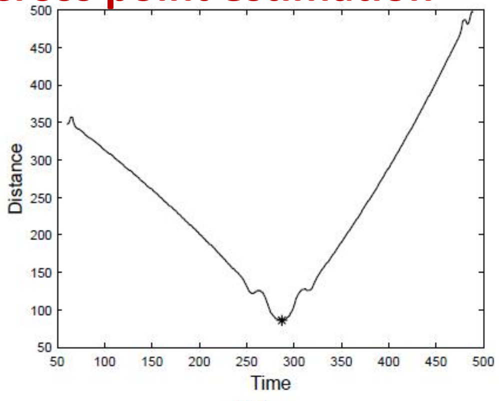
Estimation points



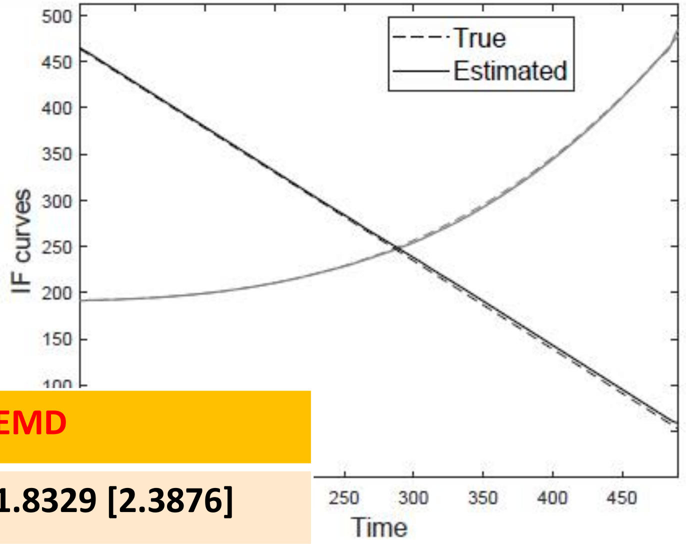
CR estimation



Cross point estimation

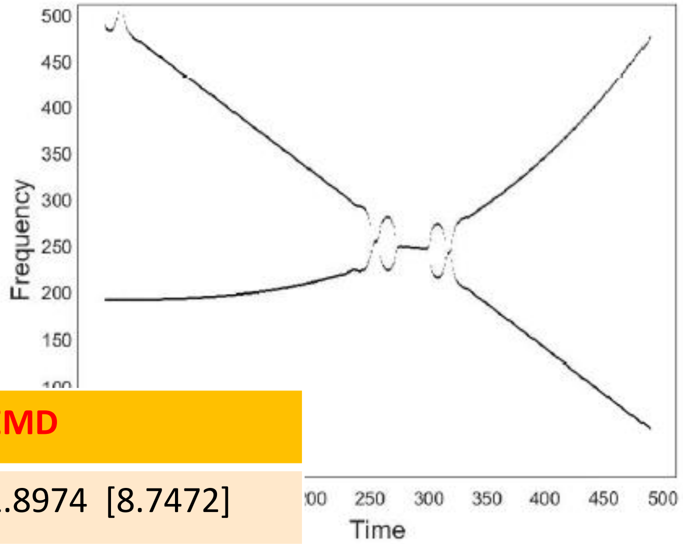


PROPOSED RESULT



EMD
1.8329 [2.3876]

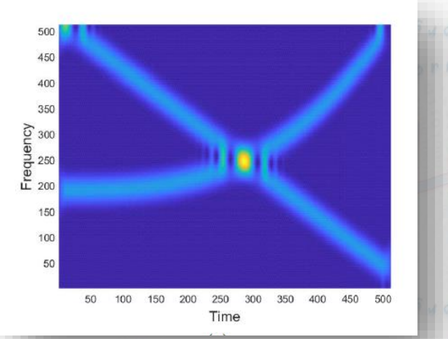
MODULUS MAXIMA

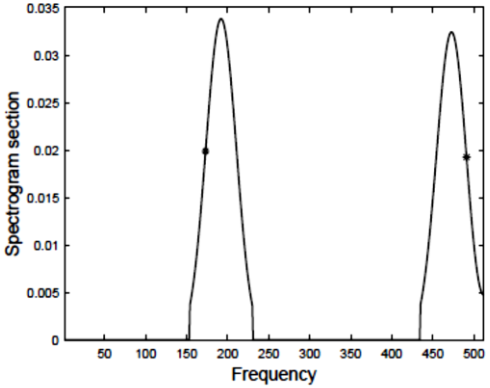


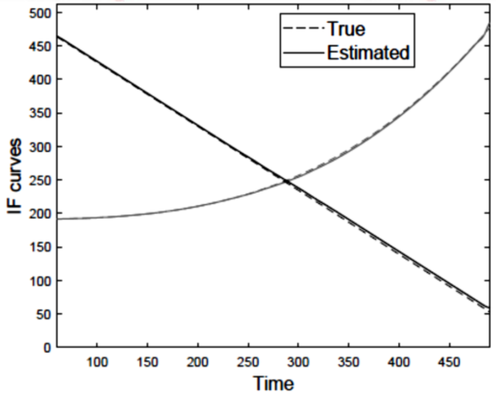
EMD
2.8974 [8.7472]

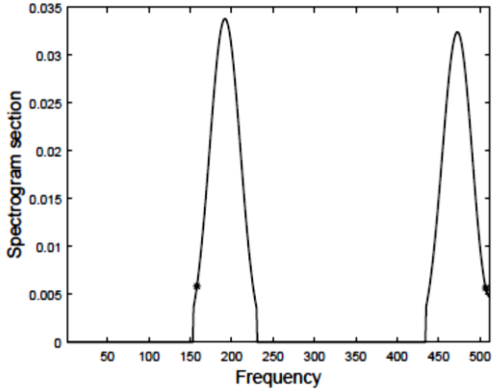
Results

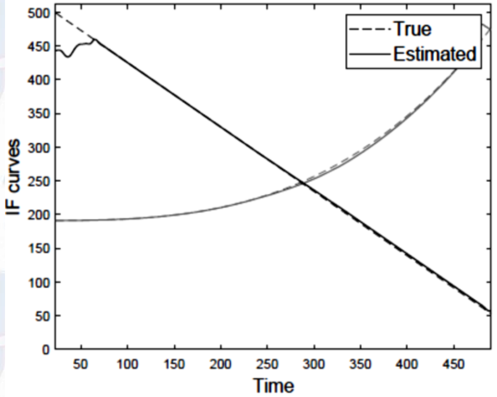
dependence on the estimation points

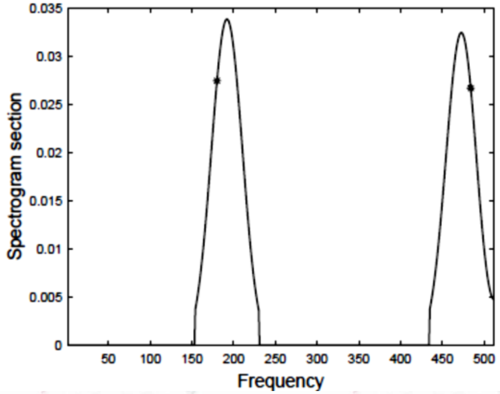


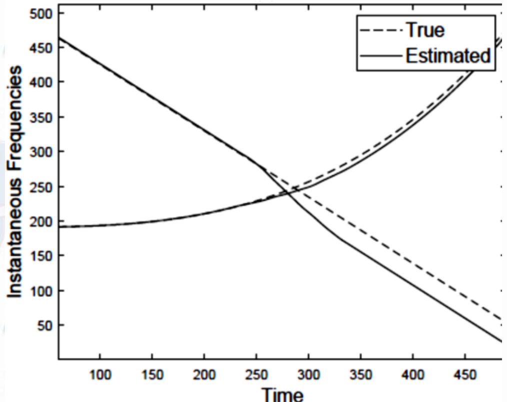












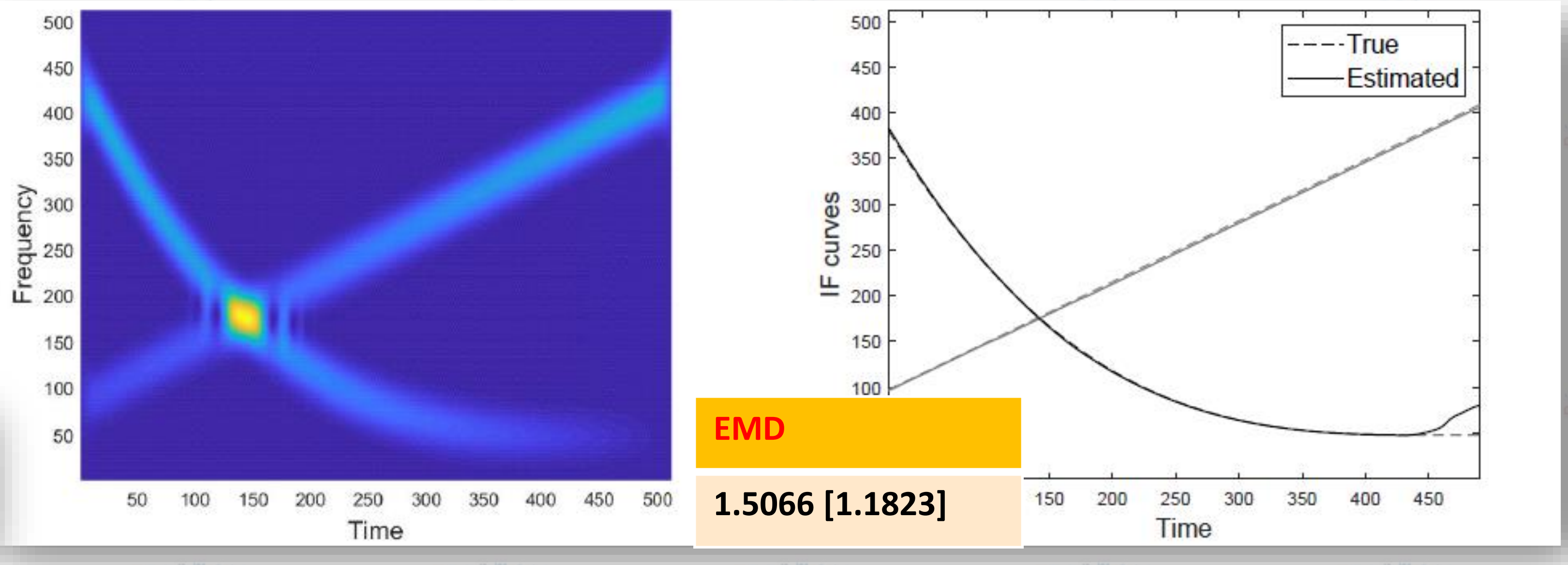
Results

robustness to noise

$$f_{12}(t) = a_{11}(t) f_1(t) + a_{21}(t) f_2(t),$$

$$a_{11}(t) = 0.5(t+1), \quad a_{21}(t) = \sqrt{1-t},$$

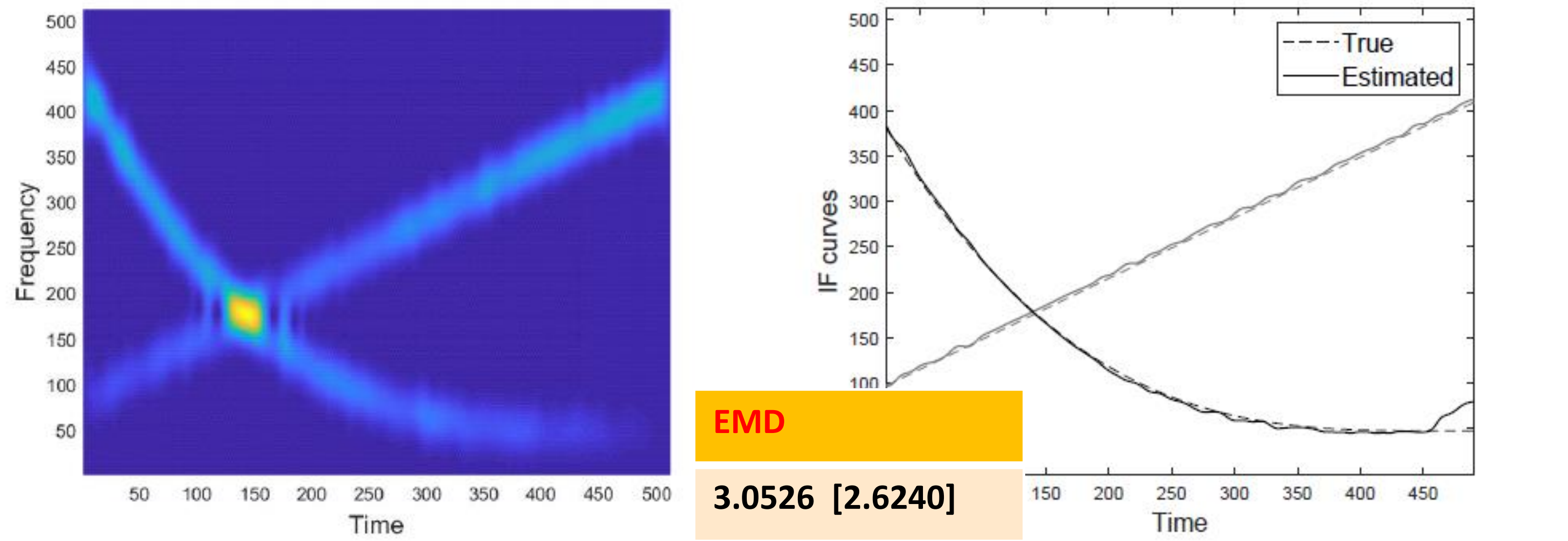
$$f_1(t) = \cos(\pi/3\, n(t+0.1)^2 + (\pi+30)t),$$
$$f_2(t) = \sin(0.26\, \pi n(0.9-t)^4 - 150t),$$



EMD

1.5066 [1.1823]

AWG noise at $SNR = 16\, dB$



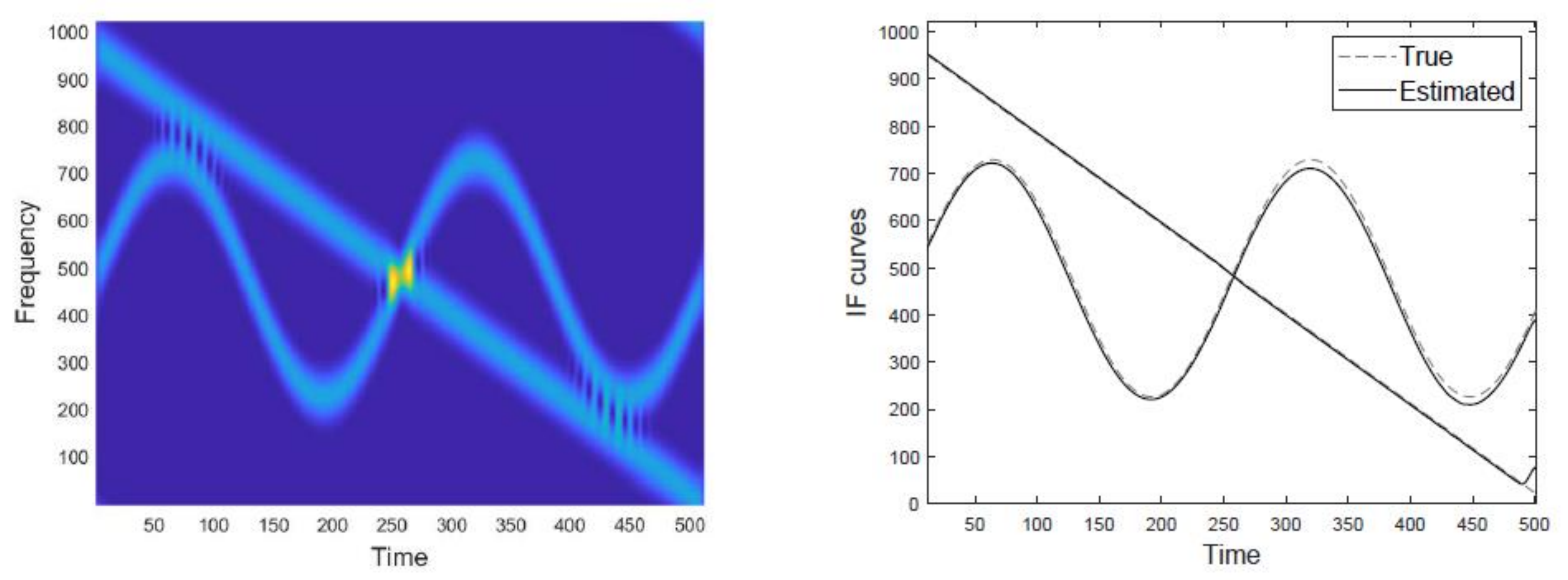
EMD

3.0526 [2.6240]

Results

robustness to IF class

$$g(t) = \exp\left[i\left(20\pi\cos(4\pi t) - 1500t\right)\right] + \exp\left[i\left(0.95\pi n t^2 + 170t\right)\right]$$



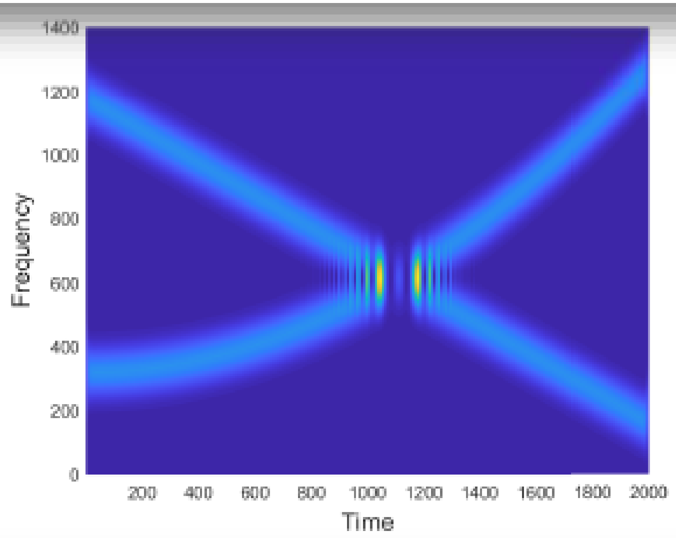
Results

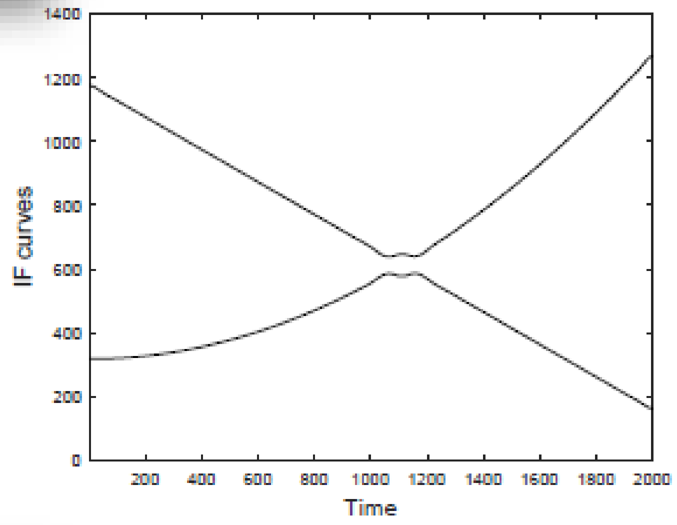
Variational based method

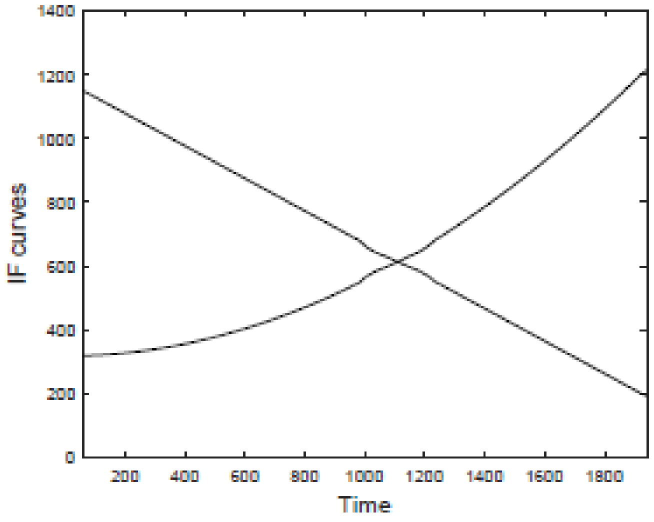
S. Chen et al., 2017,
IEEE Transactions on Signal Processing

Proposed method

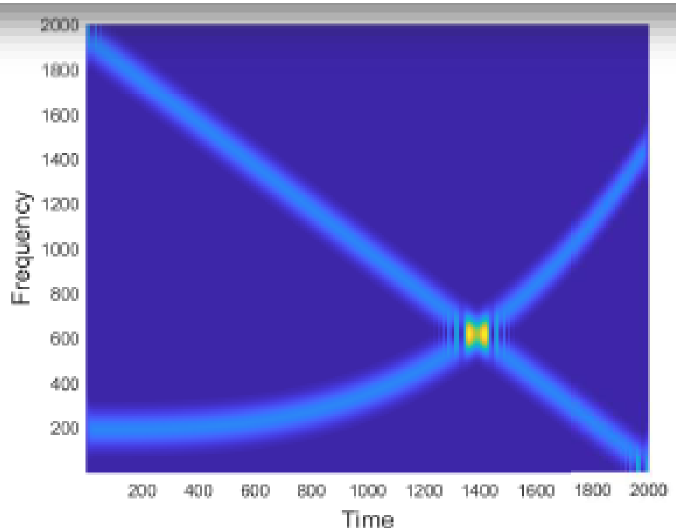
$$g_1(t) = \cos(0.5\,n\,t^3 + 1000t) + \cos(0.8\,n\,(1-t)^2 + 500(1-t))$$

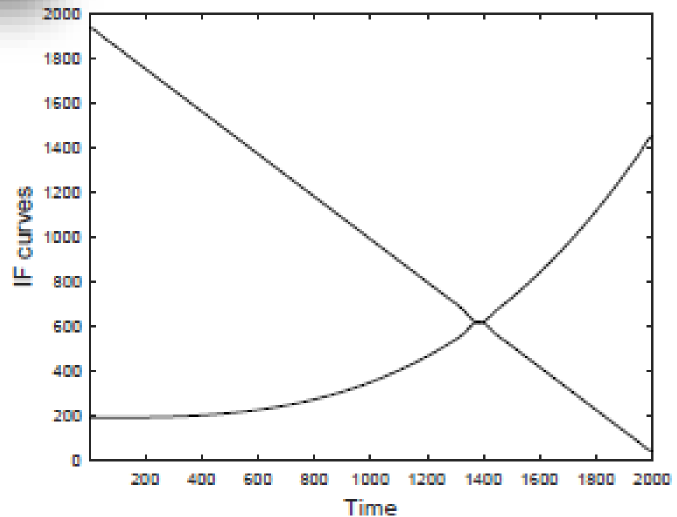


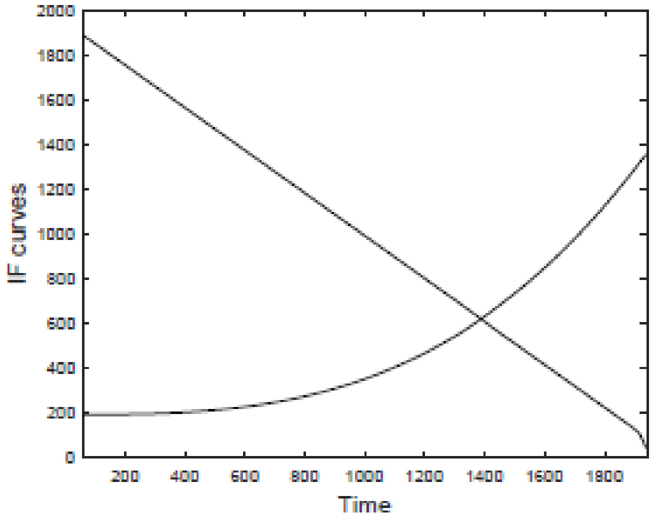




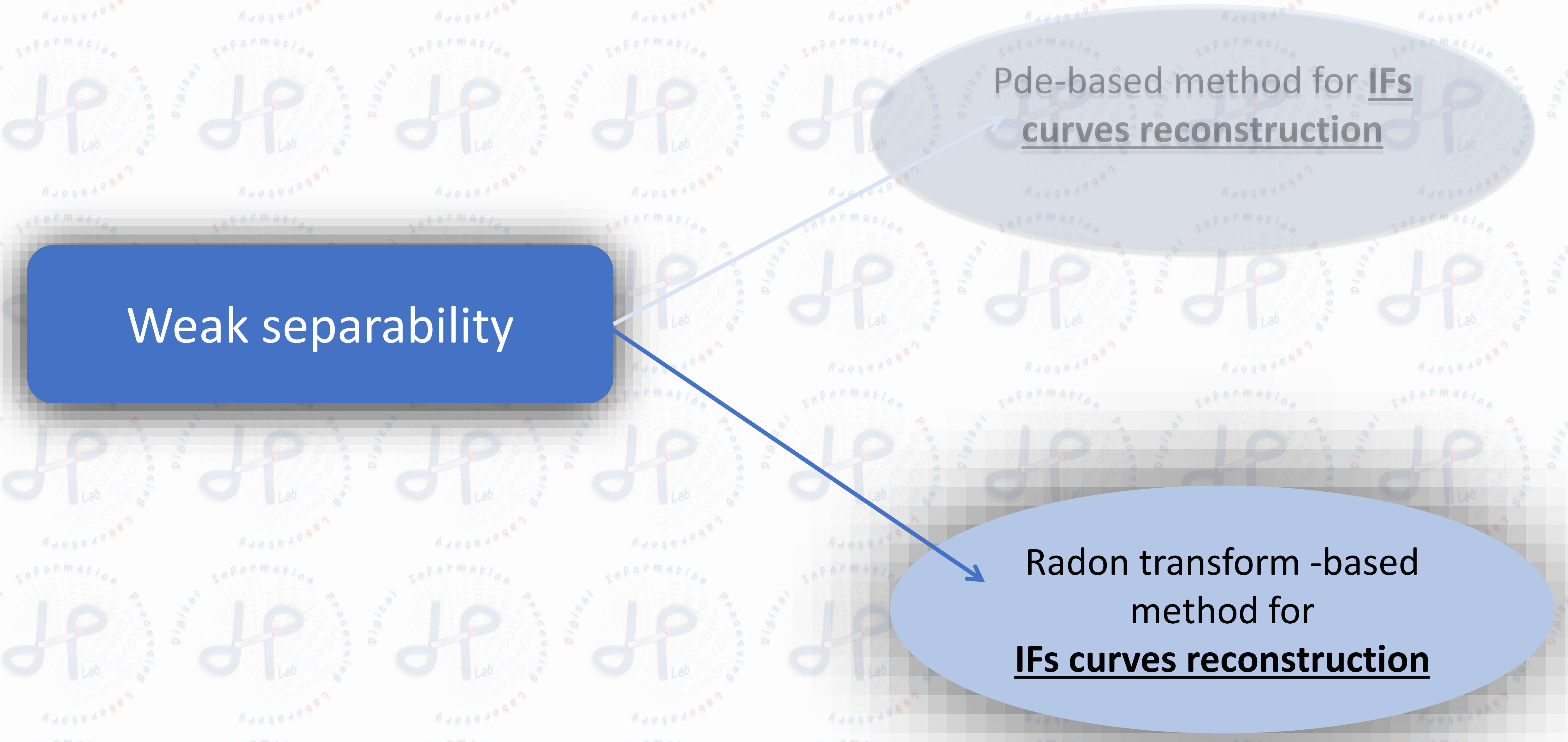
$$g_2(t) = \cos(0.5\,n\,t^4 + 600t) + \cos(1.5\,n\,(1-t)^2 + 100(1-t)),$$







Weak separability and IF curves estimation

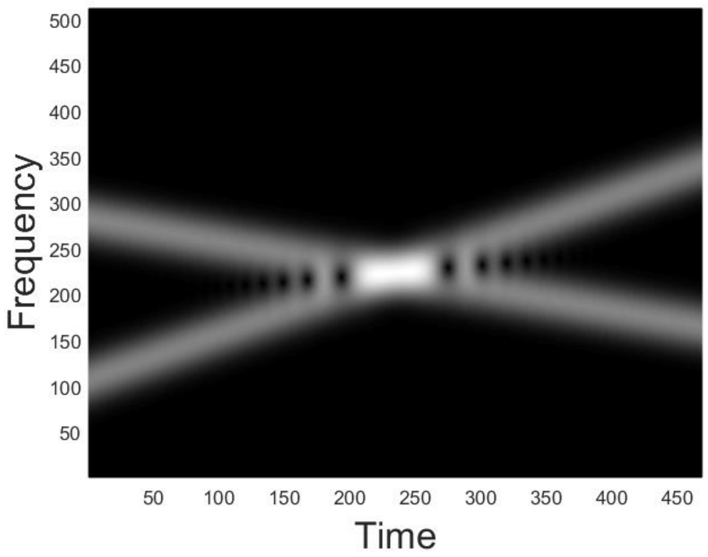


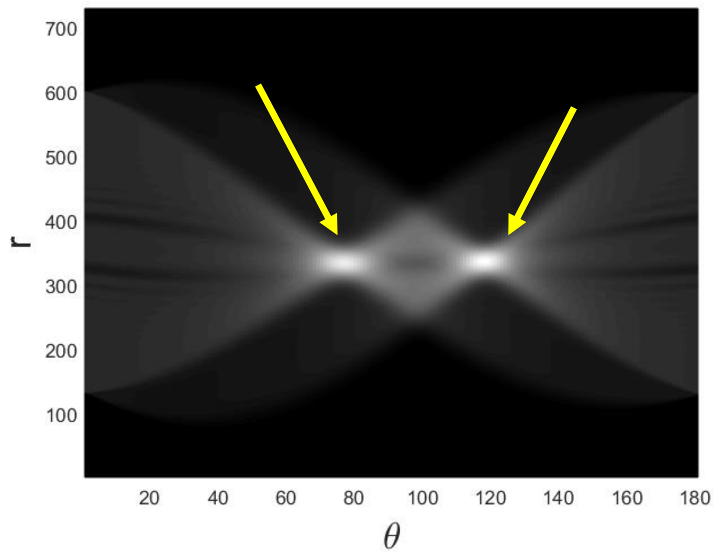
Radon-Spectrogram Distribution

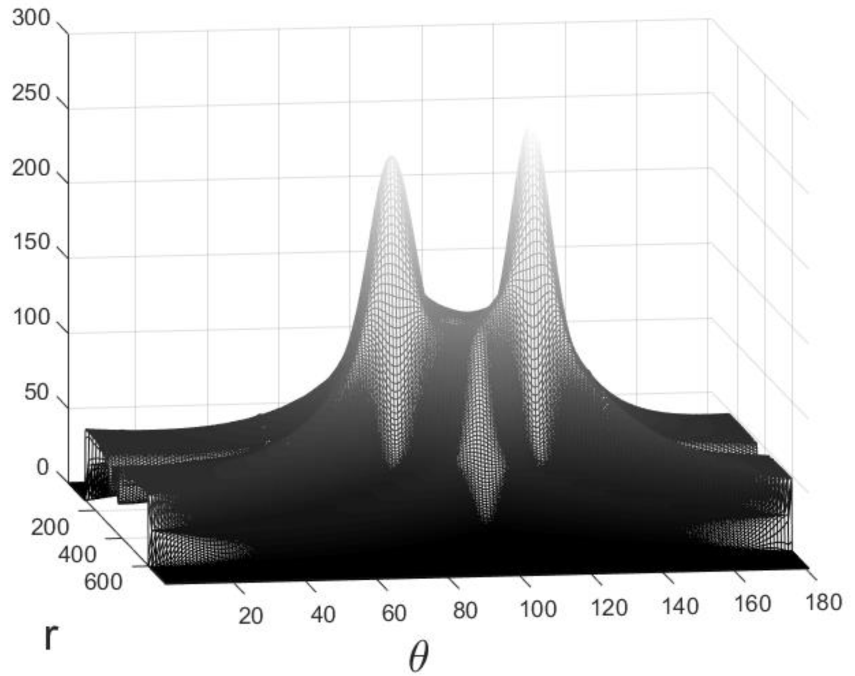
Main observation: <u>not separated modes are separable in the Radon domain</u>

TF DOMAIN → **RADON DOMAIN (view from above)**

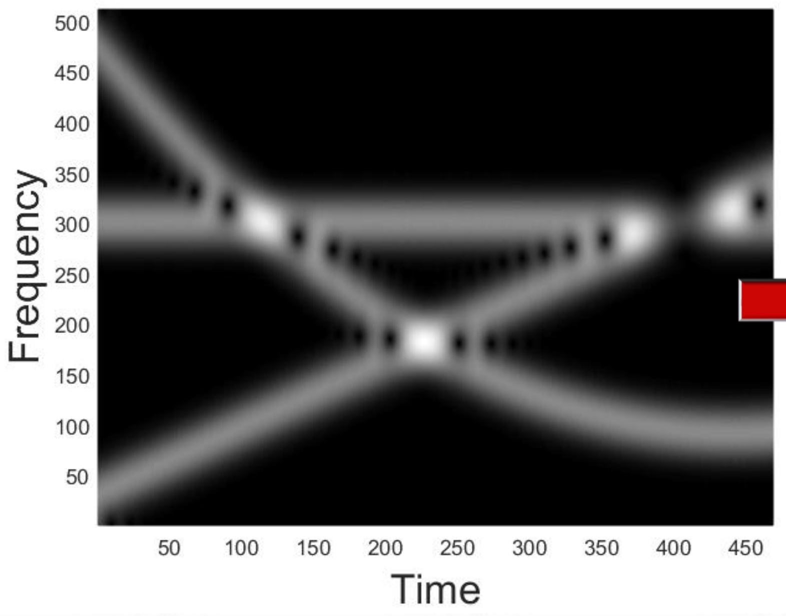
RADON DOMAIN (3D view)



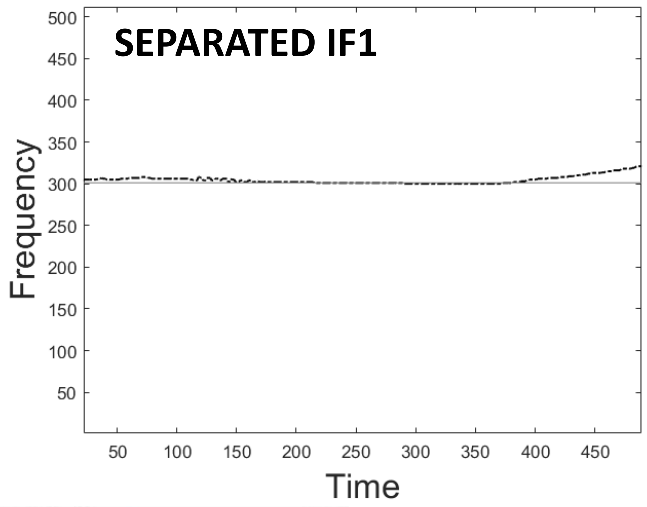


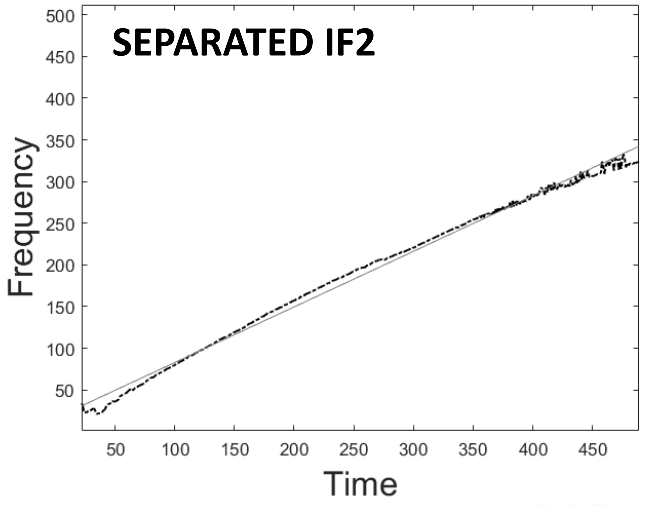


Results

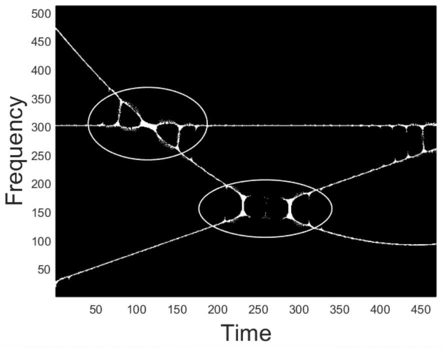


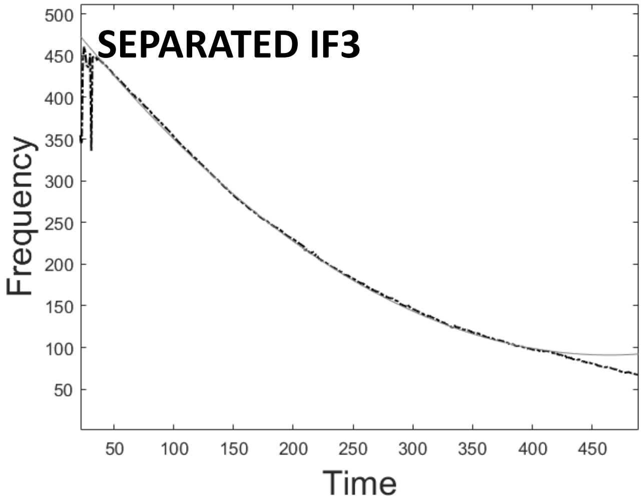
PROPOSED METHOD





REASSIGNED SPECTROGRAM using conventional methods





<u>**benefits:**</u> robustness to interference and noise