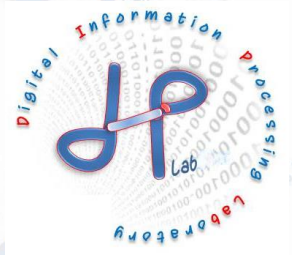
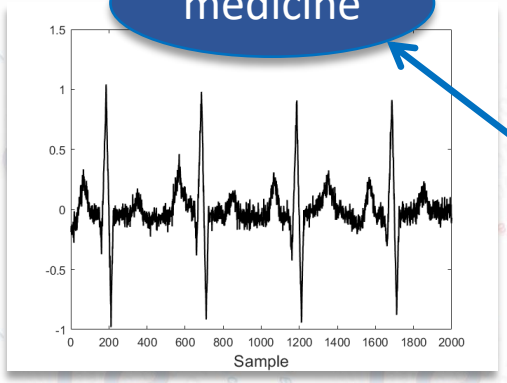
Time-frequency reassignment@DIPLab



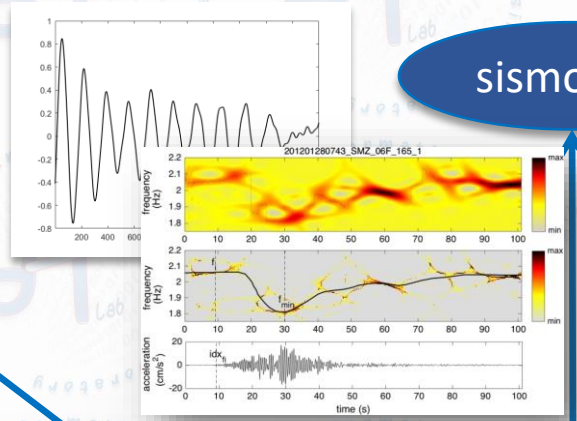
Amplitude and Frequency Modulated signals everywhere

AM-FM signals

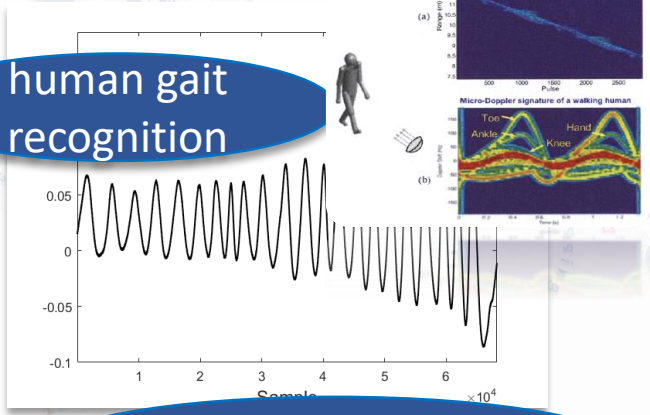
medicine



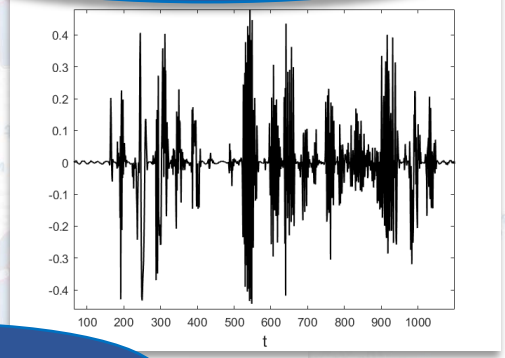
sismology



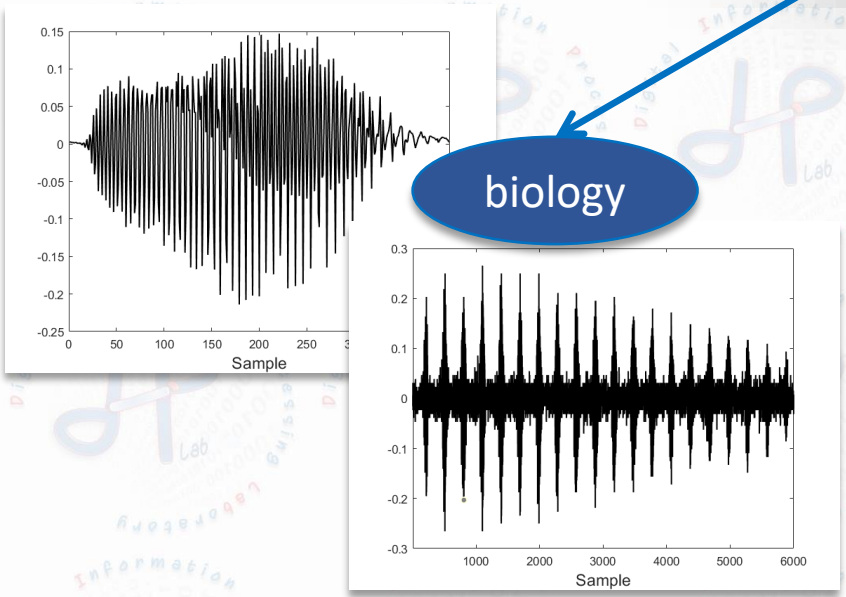
human gait recognition



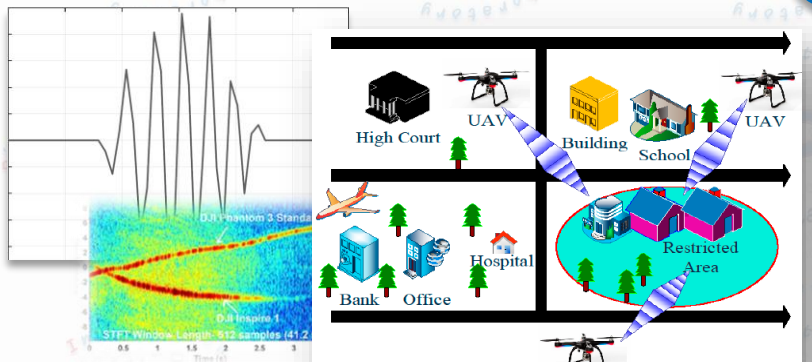
speech processing



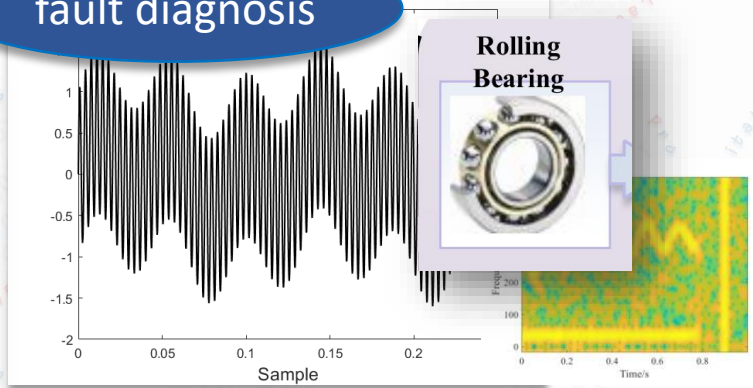
biology



airline traffic control

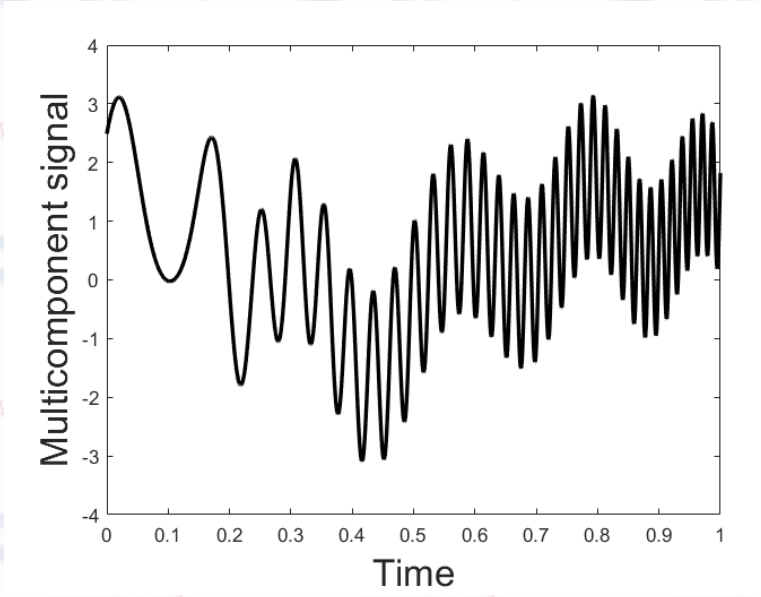


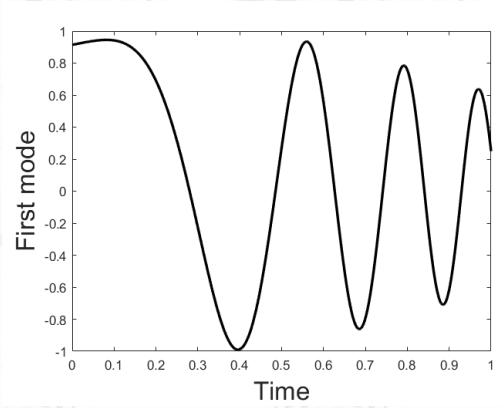
fault diagnosis

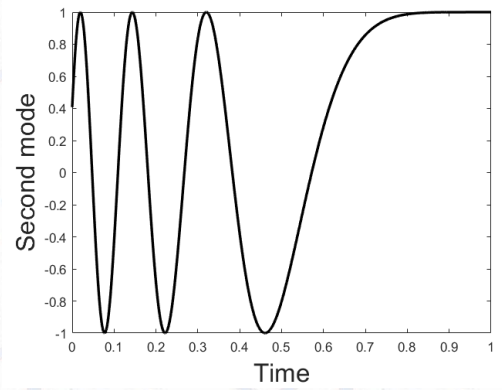


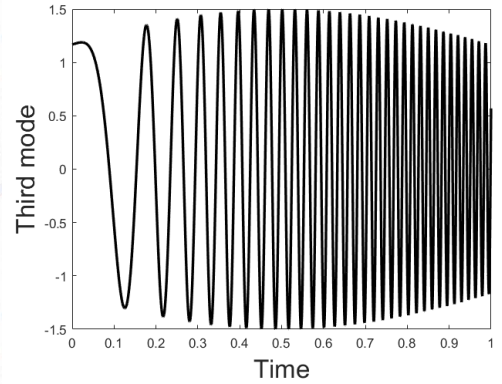
Instantaneous Frequency (IF)

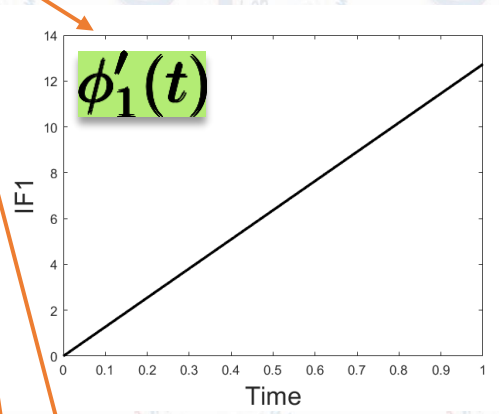
$$f(t) = \sum_{k=1}^{N} f_k(t) = \sum_{k=1}^{N} a_k(t) e^{i\phi_k(t)}$$

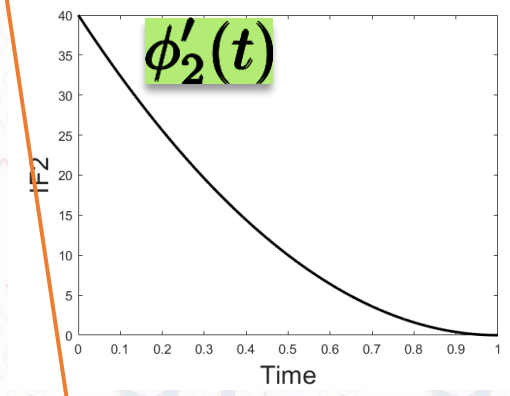


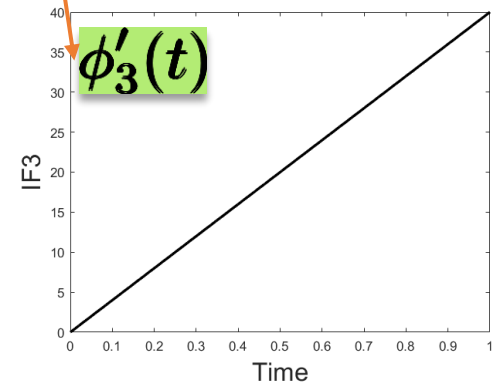






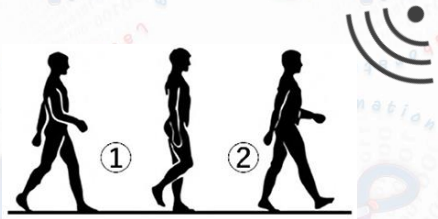


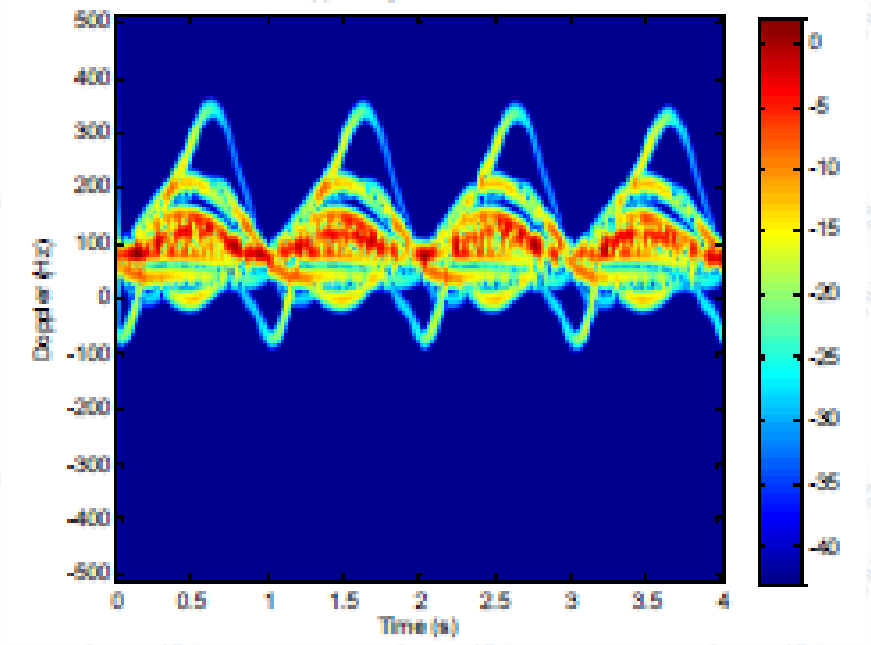




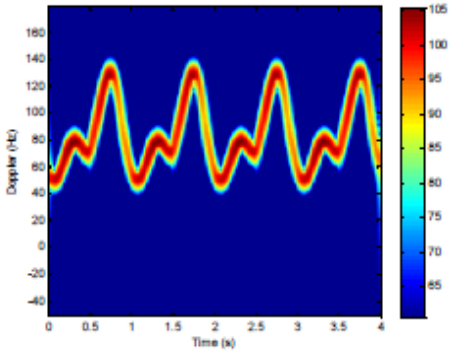
Instantaneous Frequency (IF): human gait recognition

Microdoppler signature of human body: walk

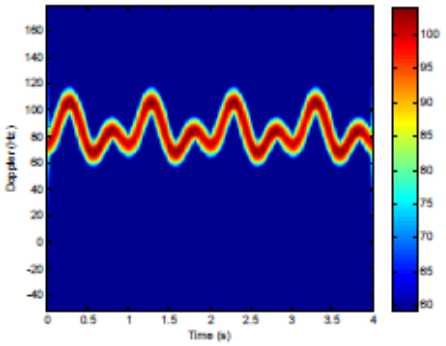




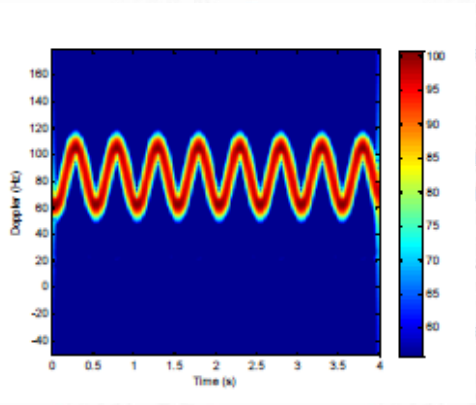
IF = speed of human body components



$\phi_1'(t)$ Leg



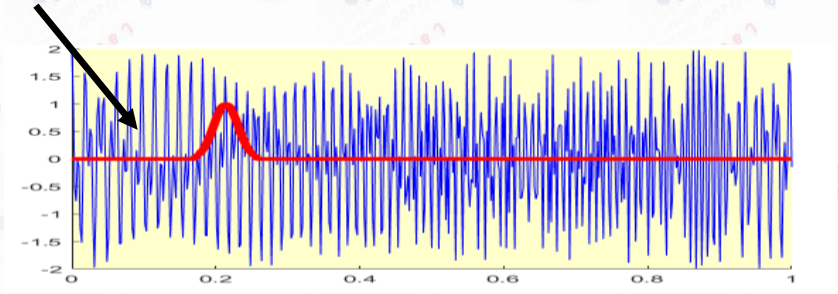
$\phi_2'(t)$ Arm

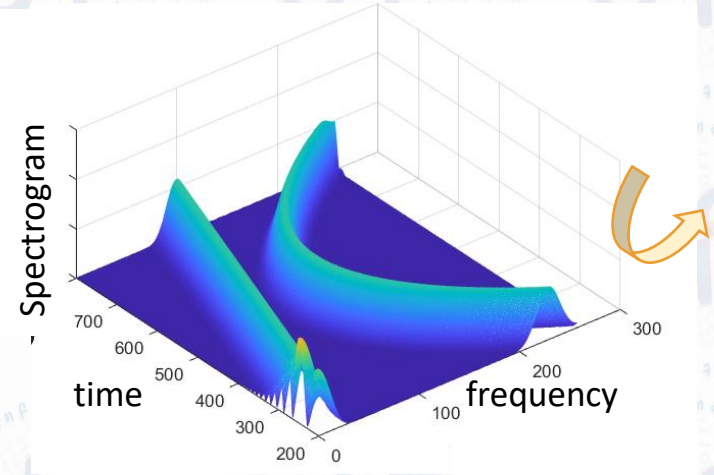


$\phi_3'(t)$ Head

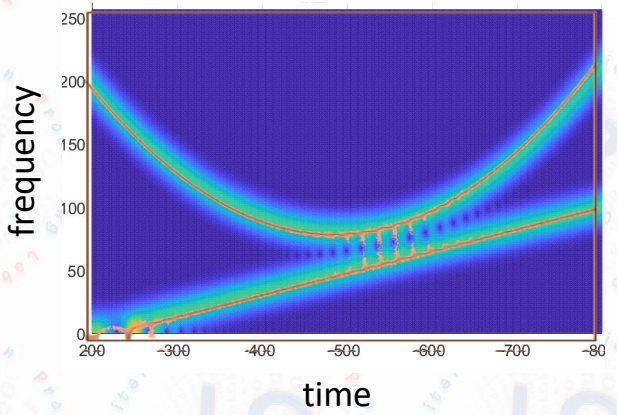
Method: time-frequency analysis

g(t)



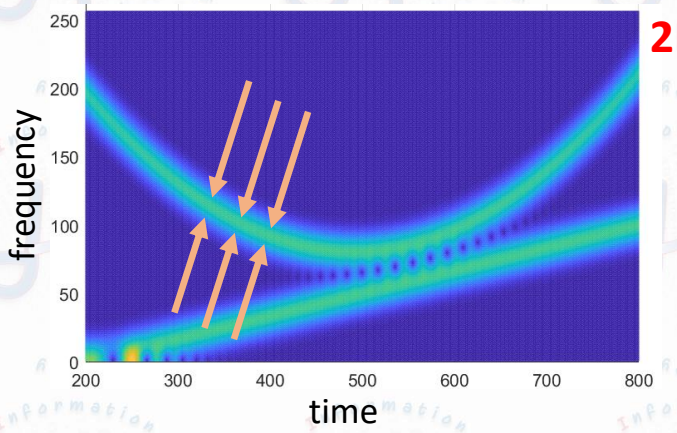


Ridge Curve $(u, \phi'(u))$



SPECTROGRAM

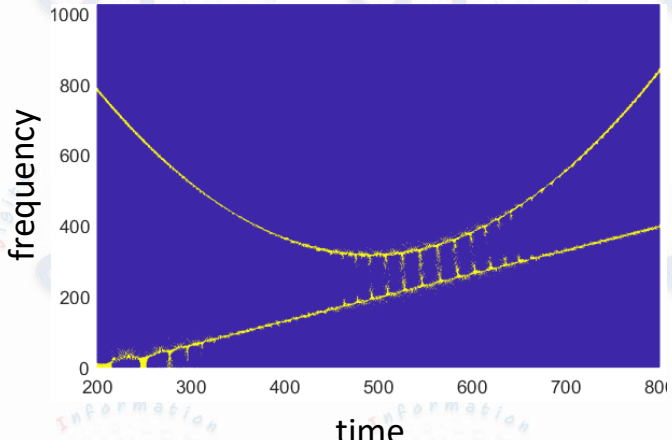
$$P_S(u,\xi)$$



2D shift

REASSIGNED SPECTROGRAM

$$\hat{P}_S(u,\xi)$$

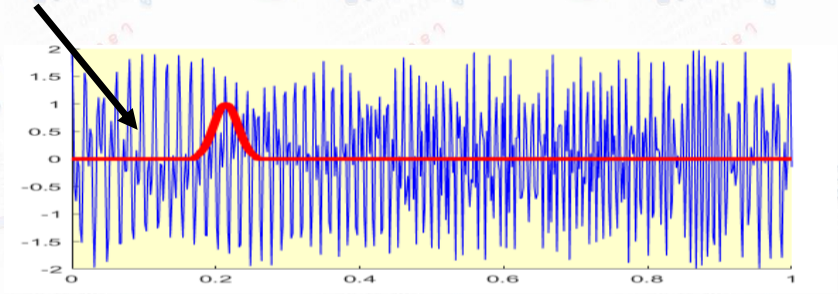


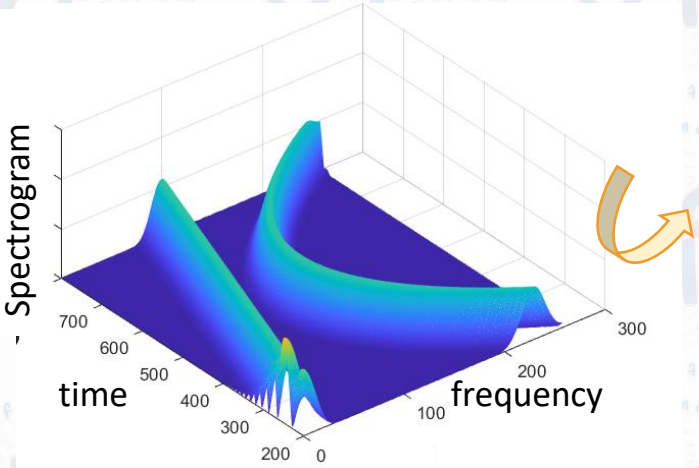
SEPARABILITY CONDITION

$$|\phi_1'(t) - \phi_2'(t)| \geq \Delta\omega$$

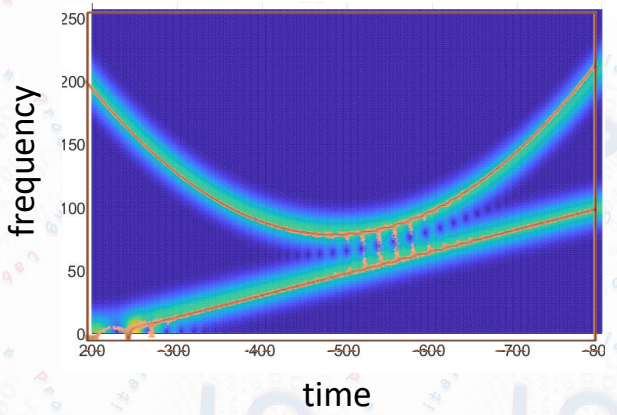
Method: time-frequency analysis

g(t)



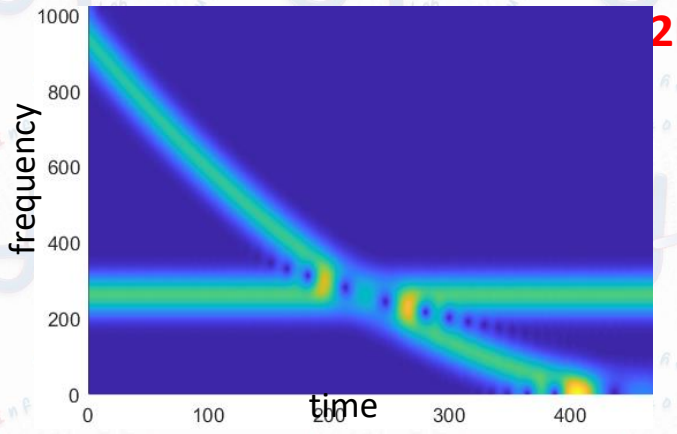


Ridge Curve $(u, \phi'(u))$



SPECTROGRAM

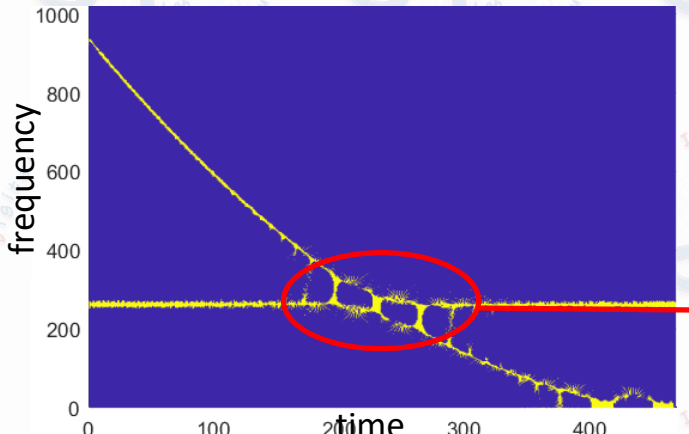
$$P_S(u,\xi)$$



2D shift

REASSIGNED SPECTROGRAM

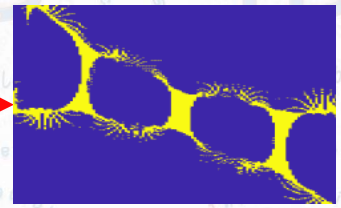
$$\hat{P}_S(u,\xi)$$



SEPARABILITY CONDITION

~~$|\phi_1'(t) - \phi_2'(t)| \geq \Delta\omega$~~

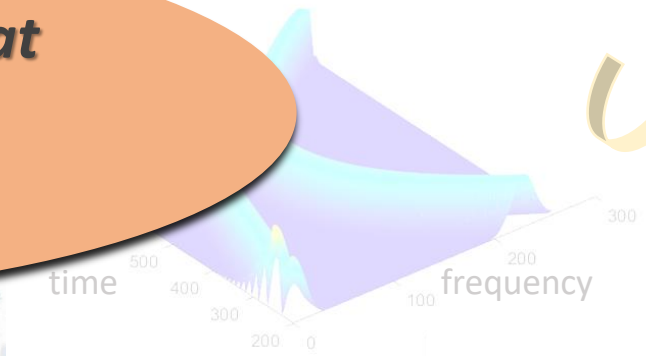
$|\phi_1'(t) - \phi_2'(t)| < \Delta\omega$



Method: time-frequency analysis

g(t)

Goal: ***Correct reassignment at each time-frequency point,*** **independently of IF class?**



Ridge Curve $(u, \phi'(u))$

SPECTROGRAM

$P_S(u,\xi)$

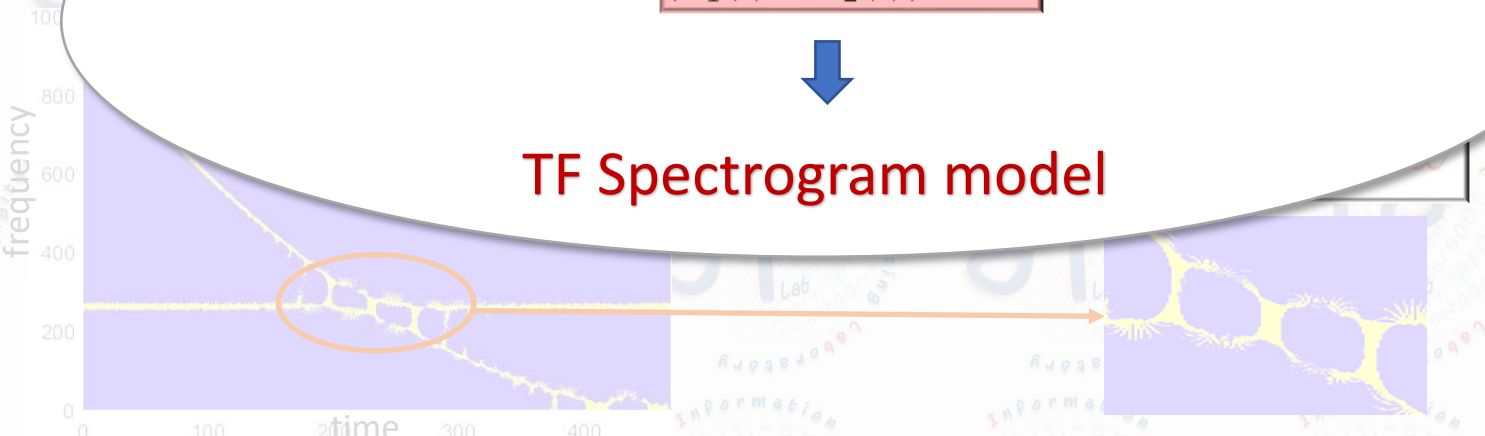
2D shift

REASSIGNED SPECTROGRAM

Weakening the separability condition

$$|\phi_1'(t) - \phi_2'(t)| \geq \Delta\omega$$

TF Spectrogram model



Main contribution

Definition of local and pointwise methods for TF analysis of frequency modulated multicomponent signals having non separable modes

⬇ *method*

spectrogram evolution law and weakened separability

⬇ *advantages*

- ✓ non parametric approaches
- ✓ independency of IF functional class
- ✓ better modes reconstruction in TF interference (non separable) region
- ✓ robustness to moderate noise

⬇ *requirements*

modes counting and *interference region detection*

Method: weak separability

Definition 1[*Separability condition*]
Two modes with IFs $\phi_1'(u)$ and $\phi_2'(u)$ are separated at time location u if

$$|\phi_1'(u) - \phi_2'(u)| \geq \Delta\omega,$$

where $\Delta\omega$ denotes the analysis window frequency bandwidth.

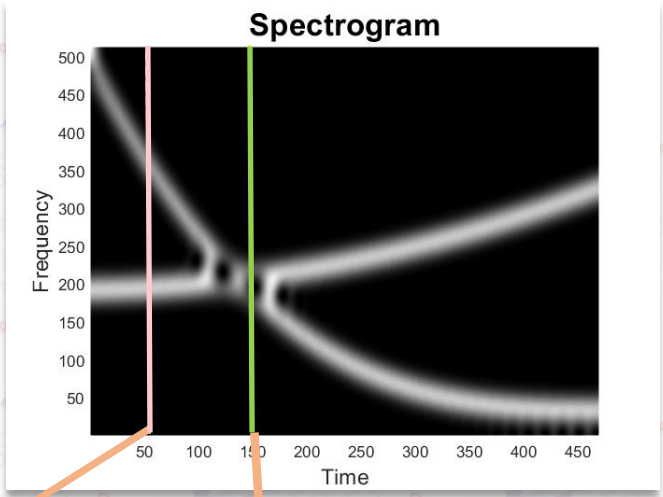
Definition 2 [*Weakened separability condition*]
Two modes with IFs $\phi_1'(u)$ and $\phi_2'(u)$ are separated at time location u if there exists at least one curve in $\mathcal{C}_{c_1,\phi_1}$, i.e., $\xi_1(u) = \phi_1'(u) + c_1$, such that

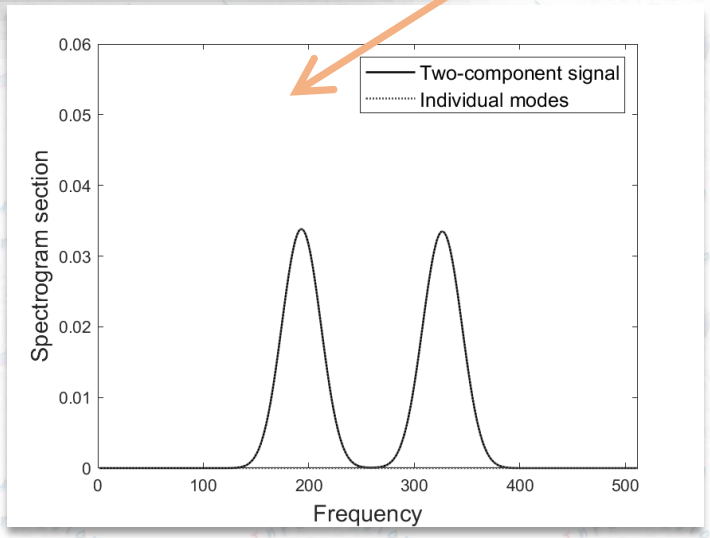
$$|\xi_1(u) - \phi_2'(u)| \geq \Delta\omega;$$

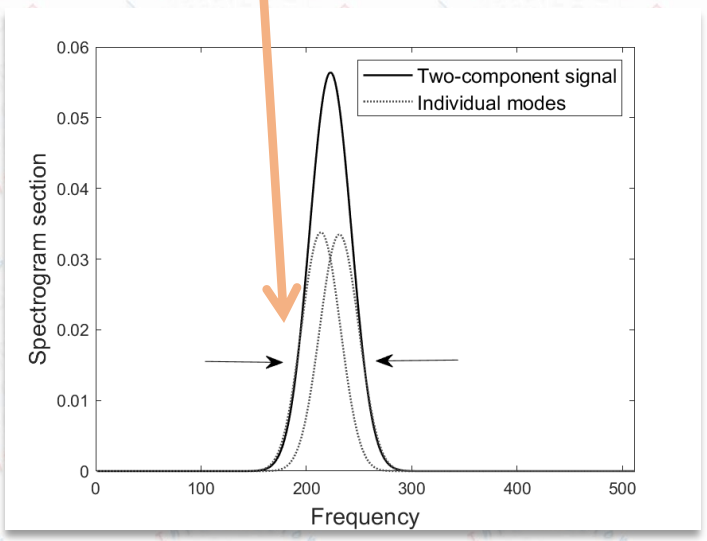
or viceversa.

Two-components pde reduces to

$$\frac{\partial P(u,\xi)}{\partial u} + \phi''(u)\frac{\partial P(u,\xi)}{\partial \xi} - \frac{2a'(u)}{a(u)}P(u,\xi) = 0$$







Weak separability and reassignment

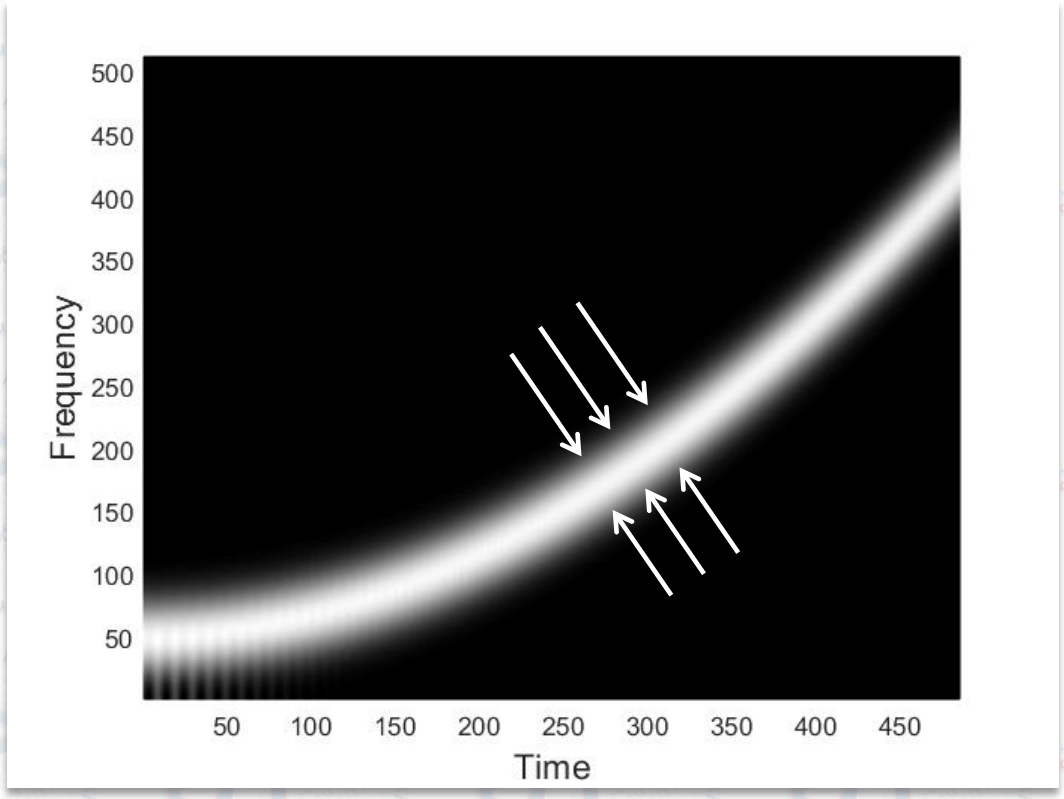
Weak separability

Iterative reassignment for **IFs curves resolution enhancement**

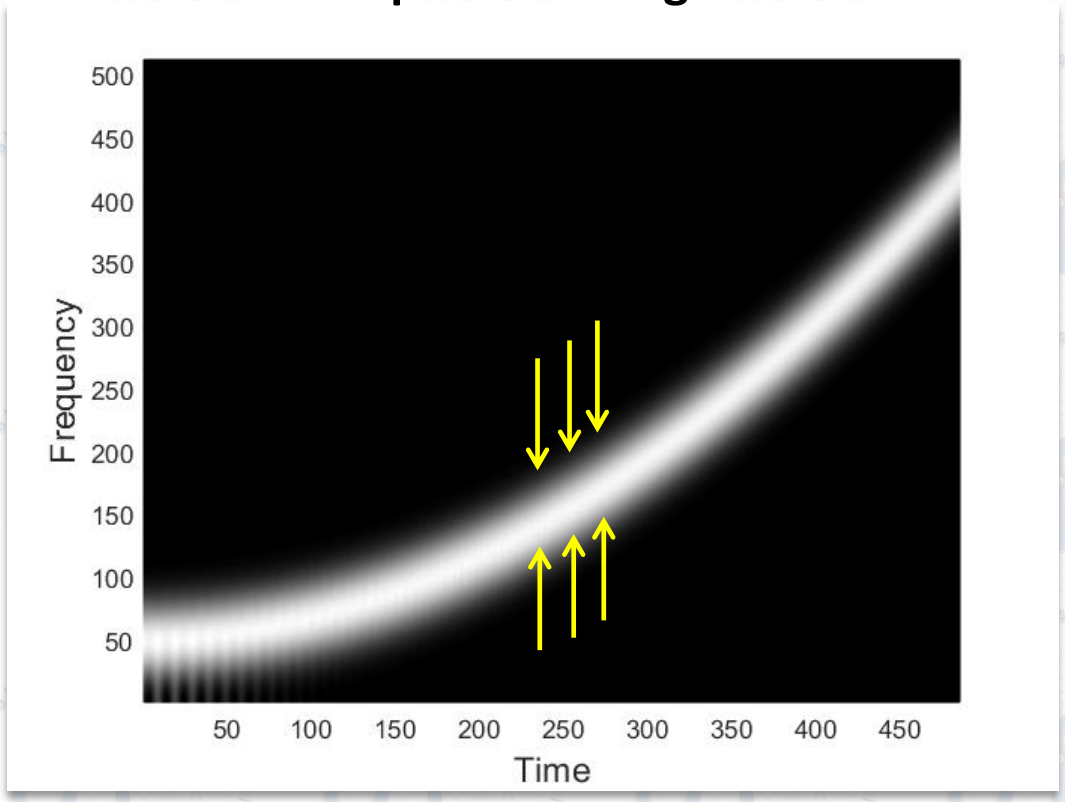
skeleton -based method for IFs curves reconstruction

Iterative spectrogram reassignment

Classical reassignment



Proposed reassignment



$$\hat{u}_f(u,\xi) = u + \Re\left(\frac{S_f^{tg}(u,\xi)}{S_f^{g}(u,\xi)}\right)$$

$$\hat{\xi}_f(u,\xi) = \xi - \Im\left(\frac{S_f^{g'}(u,\xi)}{S_f^{g}(u,\xi)}\right)$$

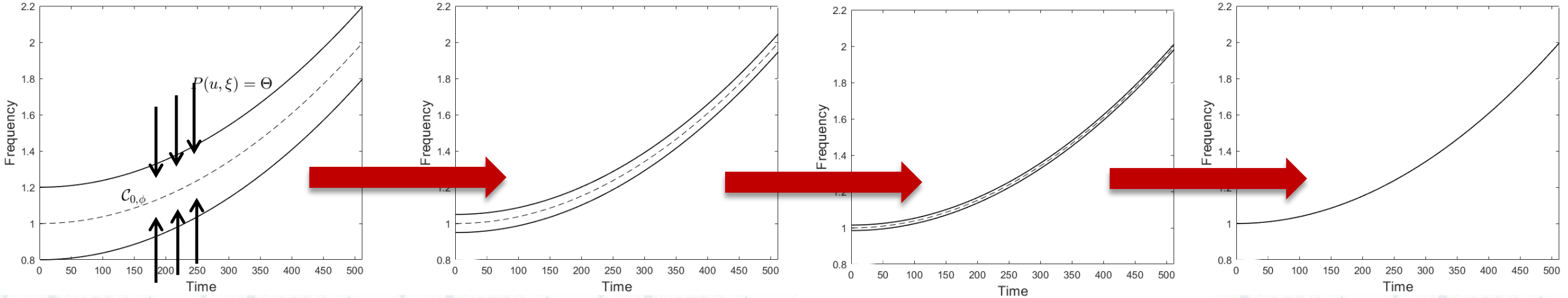
$$\frac{\partial P(u,\xi)}{\partial \xi} = 0$$

Newton-like method

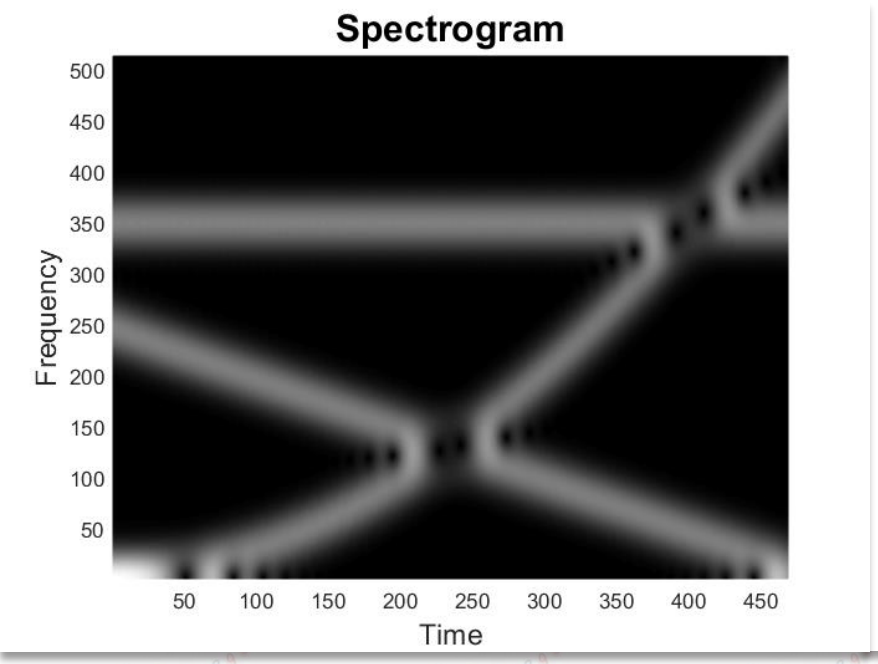
Iterative spectrogram reassignment

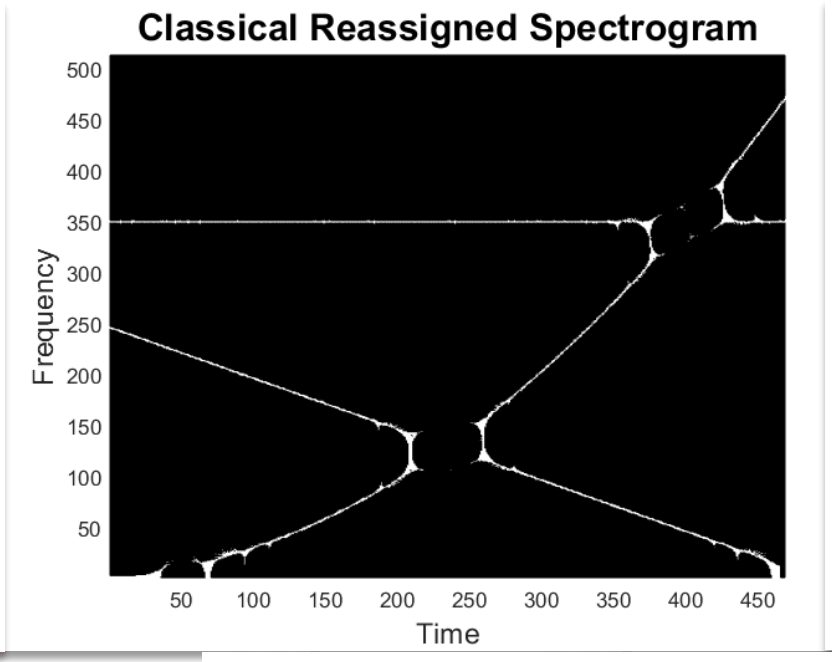
$$\xi_{k+1} = \xi_k \pm \Delta_k(P(u, \xi_k), g),\ k \geq 0,\ \text{fixed } u$$

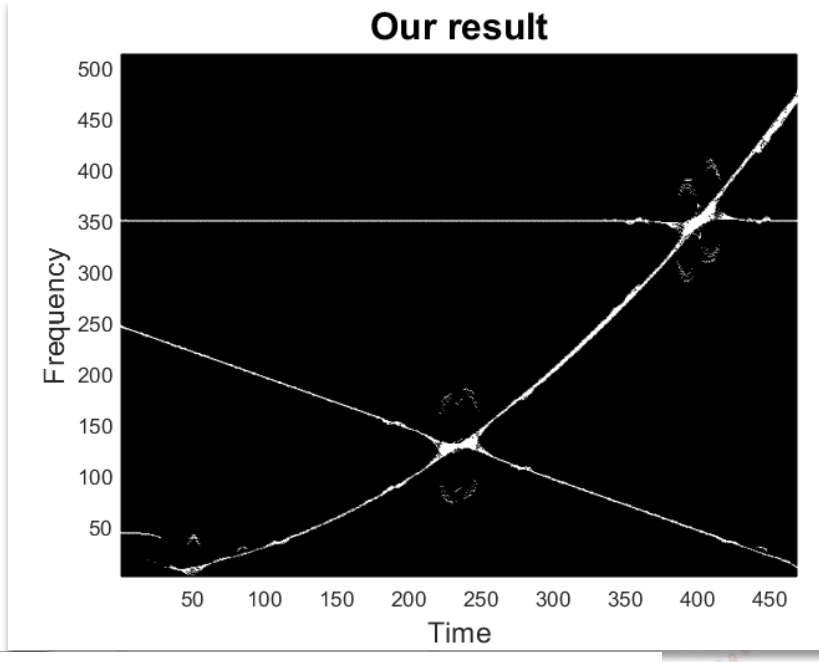
the shift Δ_k is **estimated** from the data



Iterative spectrogram reassignment







Earth Mover's Distance (*EMD*) - Discrete Wasserstein distance

Reassignment	Proposed method
66.79	**25.45**
74.71	**27.53**

benefits: computational effort (convergence reached in 2-3 iterations, only 1 STFT required), robustness to interference

issues: limited to constant amplitude signals

Weak separability and reassignment

Weak separability

Iterative reassignment for IFs curves resolution enhancement

Skeleton-based method for **IFs curves resolution enhancement**